PROJECT MANAGEMENT

Critical Path Method-- Program, Evaluation & Review Technique (CPM/PERT)

As Marketing requested it
As Sales ordered it
As Engineering designed it
As Data Processing programmed it
As Services installed it
What the Customer ordered
Introduction to Project Management

Many people and organizations today have a new or renewed interest in project management. Until the 1980s, project management primarily focused on providing schedule and resource data to top management in the military, computer, and construction industries. Today's project management involves much more, and people in every industry and every country manage projects. New technologies have become a significant factor in many businesses. Today's companies, governments, and non-profit organizations are recognizing that to be successful, they need to be conversant with and use modern project management techniques. Individuals are realizing that to remain competitive in the workplace, they must develop skills to become good project team members and project managers. They also realize that many of the concepts of project management will help them in their everyday lives as they work with people and technology on a day-to-day basis.

Although people have worked on projects for centuries, most agree that the modern concept of project management began with the Manhattan Project, which the U.S. military led to develop the atomic bomb. The Manhattan Project involved many people with different skills at several different locations. It also clearly separated the overall management of the project's mission, schedule, and budget under General Leslie R. Groves and the technical management of the project under Dr. Robert Oppenheimer. The Manhattan Project lasted about three years and cost almost $2 billion in 1946. In developing the project, the military realized that scientists and other technical specialists often did not have the desire or the necessary skills to manage large projects. For example, after being asked several times for each team member's responsibilities at the new Los Alamos laboratory in 1943, Dr. Oppenheimer threw a piece of paper with an organization chart on it at his director and said, "Here's your damn organization chart." Project management was recognized as a distinct discipline requiring people with special skills and, even more importantly, the desire to lead project teams.

WHAT IS A PROJECT?

Project management was first used to manage the US space program. Its practice has now been expanded rapidly through the government, the military and the corporate world. A project is "a temporary endeavour undertaken to create a unique product, service, or result." Operations, on the other hand, is work done in organizations to sustain the business. Projects are different from operations in that they end when their objectives have been reached or the project has been terminated.

a.) Definition of a Project
The Project Management Institute defines a project as follows:

A Project is a unique undertaking with a defined starting point and duration directed at achieving defined objectives, utilizing finite or infinite resources.

The key parts of this definition:
1. A project has a unique objective.
2. A project has a definite start, duration and finish. It has a temporary rather than open-ended duration.

Some examples of projects are:
- Building a house
- Relocating a data center
- Writing a book
- Developing a software program

b.) Project Management
Project management is the management of an organized set of activities directed toward a common goal, using specialized management structures and techniques. It includes:
• **Determining project objectives**
  What is the goal (or goals) of the project? Examples of project goals include building a bridge, relocating the MIS department to a new site or installing a new phone system. More importantly, some examples of things that are NOT projects include scheduling the usage for a training facility or scheduling engineers in a technical service department. These are not projects because they do not meet all the criteria of a project. They do not have a definitive start, finish, and duration.

• **Managing budgets and resources**
  Projects do not get done without resources to do them. To ensure successful completion of a project, it is important to estimate correctly the number of personnel and the amount of equipment needed. With this, it is important to realize the cost of the project. Some projects can be completed in a shorter time by increasing the manpower on the project. However, doing this also increases the cost. One of the project manager's jobs is to maintain a balance between reducing costs and reducing the time to complete the project.

• **Reporting Progress**
  Reporting progress is a key to project management. It is essential that key players in a project know what is happening, and whether they are on track, behind, or ahead of schedule. By reviewing progress on a regular basis, you can try to avoid possible problems in advance. For example, if you notice that a certain task was scheduled to take 10 days to accomplish, but on day 5 only 25% of the work was finished, you could possibly re-allocate resources to that task in order to complete it on time.

• **Evaluating efficiency and effectiveness**
  During and after a project, it is important to review and analyze the performance on the project. This information can provide valuable insight into possible changes to make for future projects. For example, your project was to build a house, and one of the steps involved was landscaping. After the project is finished, you notice that it took less time to do the landscaping than you originally planned. This information could be valuable if you build another house, because you could reduce the time allocated for landscaping. By constantly reviewing the efficiency and effectiveness of your project, you can more accurately plan future projects.

Here is the main definition of what project management is:
  1. Project management is no small task.
  2. Project management has a definite beginning and end. It is not a continuous process.
  3. Project management uses various measurement tools to accomplish and track project tasks. These include Gantt and Pert charts.
  4. Projects frequently need resources on an add-on basis as opposed to organizations who have full-time positions.

There are three main points that are most important to a successful project:
  1. A Project must meet customer requirements.
  2. A Project must be under budget.
  3. A Project must be on time.

There are four phases a project goes through. The role of the project manager in project management is one of great responsibility. It's the project manager's job to direct and supervise the project from beginning to end. Here are some other roles:
  1. The project manager must define the project, reduce the project to a set of manageable tasks, obtain appropriate and necessary resources, and build a team or teams to perform the project work.
  2. The project manager must set the final goal for the project and must motivate his workers to complete the project on time.
  3. A project manager must have is technical skills. This relates to financial planning, contract management, and managing creative thinking and problem solving techniques are promoted.
  4. No project ever goes 100% as planned, so project managers must learn to adapt to change.
There are many things that can go wrong with project management. These are commonly called barriers. Here are some possible barriers:

1. Poor Communication
   - Many times a project may fail because the project team does not know exactly what to get done or what's already been done.

2. Disagreement
   - Project must meet all elements in a contract.
   - Customer and project manager must agree on numerous elements.

3. Failure to comply with standards and regulations.

4. Inclement weather.

5. Union strikes.

6. Personality conflicts.

7. Poor management

8. Poorly defined project goals

c.) Project Managers

Project managers are responsible for managing projects. They coordinate projects and related tasks, but do not usually have direct management responsibilities for resources assigned to their project. The resources involved in one project may not be the same resources involved in another project. Project managers focus only on work that is specific to their project, and are primarily task and time-constrained: “How do I ensure my project gets finished in the shortest amount of time?” is a question on every project manager’s mind each day.

An effective project manager is crucial to a project’s success. Project managers work with the project sponsors, the project team, and the other people involved in a project to meet project goals. Project managers should also possess general management knowledge and skills. They should understand important topics related to financial management, accounting, procurement, sales, marketing, contracts, manufacturing, distribution, logistics, the supply chain, strategic planning, tactical planning, operations management, organizational structures and behaviour, personnel administration, compensation, benefits, career paths, and health and safety practices. On some projects, it will be critical for the project manager to have a lot of experience in one or several of these general management areas. On other projects, the project manager can delegate detailed responsibility for some of these areas to a team member, support staff, or even a supplier. Even so, the project manager must be intelligent and experienced enough to know which of these areas are most important and who is qualified to do the work. He or she must also make and/or take responsibility for all key project decisions. Achieving high performance on projects requires soft skills, otherwise called human relations skills. Some of these soft skills include effective communication, influencing the organization to get things done, leadership, motivation, negotiation, conflict management, and problem solving. Why do project managers need good soft skills? One reason is that to understand, navigate, and meet stakeholders’ needs and expectations, project managers need to lead, communicate, negotiate, solve problems, and influence the organization at large. They need to be able to listen actively to what others are saying, help develop new approaches for solving problems, and then persuade others to work toward achieving project goals. Project managers must lead their project teams by providing vision, delegating work, creating an energetic and positive environment, and setting an example of appropriate and effective behaviour.

Project managers must focus on teamwork skills to employ people effectively. They need to be able to motivate different types of people and develop esprit de corps within the project team and with other project stakeholders. Since most projects involve changes and trade-offs between competing goals, it is important for project managers to have strong coping skills as well. It helps project managers maintain their sanity and reduce their stress levels if they cope with criticism and constant change. Project managers must be flexible, creative, and sometimes patient in working toward project goals; they must also be persistent in making project needs known.

Lastly, project managers must be able to make effective use of technology as it relates to the specific project. Making effective use of technology often includes special product knowledge or experience with a
particular industry. Project managers must make many decisions and deal with people in a wide variety of disciplines, so it helps tremendously to have a project manager who is confident in using the special tools or technologies that are the most effective in particular settings. They do not normally have to be experts on any specific technology, but they have to know enough to build a strong team and ask the right questions to keep things on track. For example, project managers for large information technology projects do not have to be experts in the field of information technology, but they must have working knowledge of various technologies and understand how the project would enhance the business. Many companies have found that a good business manager can be a very good information technology project manager because they focus on meeting business needs and rely on key project members to handle the technical details. All project managers should continue to develop their knowledge and experience in project management, general management, soft skills, and the industries they support.

**PROJECT ATTRIBUTES**

Project management is “the application of knowledge, skills, tools and techniques to project activities to meet project requirements.” Project managers must not only strive to meet specific scope, time, cost, and quality goals of projects, they must also facilitate the entire process to meet the needs and expectations of the people involved in or affected by project activities. Projects come in all shapes and sizes. The following attributes help to define a project further:

- **A project has a unique purpose.** Every project should have a well-defined objective. For example, The Director of the Project Management Office in the opening case, might sponsor an information technology collaboration project to develop a list and initial analysis of potential information technology projects that might improve operations for the company. The unique purpose of this project would be to create a collaborative report with ideas from people throughout the company. The results would provide the basis for further discussions and projects. As in this example, projects result in a unique product, service, or result.

- **A project is temporary.** A project has a definite beginning and a definite end. In the information technology collaboration project, Anne might form a team of people to work immediately on the project, and then expect a report and an executive presentation of the results in one month.

- **A project is developed using progressive elaboration.** Projects are often defined broadly when they begin, and as time passes, the specific details of the project become clearer. Therefore, projects should be developed in increments. A project team should develop initial plans and then update them with more detail based on new information. For example, suppose a few people submitted ideas for the information technology collaboration project, but they did not clearly address how the ideas would support the business strategy of improving operations. The project team might decide to prepare a questionnaire for people to fill in as they submit their ideas to improve the quality of the inputs.

- **A project requires resources, often from various areas.** Resources include people, hardware, software, or other assets. Many projects cross departmental or other boundaries to achieve their unique purposes. For the information technology collaboration project, people from information technology, marketing, sales, distribution, and other areas of the company would need to work together to develop ideas. The company might also hire outside consultants to provide input. Once the project team has selected key projects for implementation, they will probably require additional hardware, software, and network resources. People from other companies—product suppliers and consulting companies—will become resources for meeting new project objectives. Resources, however, are limited. They must be used effectively to meet project and other corporate goals.

- **A project should have a primary customer or sponsor.** Most projects have many interested parties or stakeholders, but someone must take the primary role of sponsorship. The project sponsor usually provides the direction and funding for the project. In this case, Anne Roberts would be the sponsor for the information technology collaboration project. Once further information technology projects are selected, however, the sponsors for those projects would be senior managers in charge of the main parts of the company affected by the projects. For example, if the vice president of sales initiates a project to improve direct product sales using the Internet, he or she might be the project sponsor.
• **A project involves uncertainty.** Because every project is unique, it is sometimes difficult to define its objectives clearly, estimate how long it will take to complete, or determine how much it will cost. External factors also cause uncertainty, such as a supplier going out of business or a project team member needing unplanned time off. This uncertainty is one of the main reasons project management is so challenging, especially on projects involving new technologies.

a.) **SMART Goals**

**What is a Goal?**

According to the New Comprehensive International Dictionary of the English Language a goal is a point toward which effort or movement is directed. The objective point that one is striving to reach

All goals should be **SMART Goals**

• **Specific**
  - Well defined
  - They are clear to anyone that has a basic knowledge of the project

• **Measurable**
  - Have some means to be able to know if the goal is obtainable or how far away completion is.

• **Agreed Upon**
  - Have agreement between the users and the project team on what goals should be

• **Realistic**
  - Looking at the resources, knowledge, and time available can the goal be accomplished

• **Time-Framed**
  - How much time is needed to accomplish the goal
  - Having too much time can affect the project performance

b.) **The Triple Constraint**

Every project is constrained in different ways by its scope, time, and cost goals. These limitations are sometimes referred to in project management as the **triple constraint.** To create a successful project, a project manager must consider scope, time, and cost and balance these three often-competing goals. He or she must consider the following:

- **Scope:** What work will be done as part of the project? What unique product, service, or result does the customer or sponsor expect from the project? How will the scope be verified?
- **Time:** How long should it take to complete the project? What is the project’s schedule? How will the team track actual schedule performance? Who can approve changes to the schedule?
- **Cost:** What should it cost to complete the project? What is the project’s budget? How will costs be tracked? Who can authorize changes to the budget?

Figure 1 illustrates the three dimensions of the triple constraint. Each area—scope, time, and cost—has a target at the beginning of the project.

Managing the triple constraint involves making trade-offs between scope, time, and cost goals for a project. For example, you might need to increase the budget for a project to meet scope and time goals. Alternatively, you might have to reduce the scope of a project to meet time and cost goals. Experienced project managers know that you must decide which aspect of the triple constraint is most important. If time is most important, you must often change the initial scope and/or cost goals to meet the schedule. If scope goals are most important, you may need to adjust time and/or cost goals.

Although the triple constraint describes how the basic elements of a project—scope, time, and cost—interrelate, other elements can also play significant roles. Quality is often a key factor in projects, as is customer or sponsor satisfaction. Some people, in fact, refer to the “**quadruple constraint**” of project management, including quality along with scope, time, and cost. Others believe that quality considerations, including customer satisfaction, must be inherent in setting the scope, time, and cost goals of a project. A project team may meet scope, time, and cost goals but fail to meet quality standards or satisfy their sponsor, if they have not adequately addressed these concerns.
Figure 1. The Triple Constraint of Project Management

Figure 2 illustrates a framework to help you understand project management. Key elements of this framework include the project stakeholders, project management knowledge areas, project management tools and techniques, and the contribution of successful projects to the enterprise.

c.) **Project Stakeholders**

**Stakeholders** are the people involved in or affected by project activities and include the project sponsor, project team, support staff, customers, users, suppliers, and even opponents of the project. These
stakeholders often have very different needs and expectations. For example, building a new house is a well known example of a project. There are several stakeholders involved in a home construction project.

- The project sponsors would be the potential new homeowners. They would be the people paying for the house and could be on a very tight budget, so they would expect the contractor to provide accurate estimates of the costs involved in building the house. They would also need a realistic idea of when they could move in and what type of home they could afford given their budget constraints. The new homeowners would have to make important decisions to keep the costs of the house within their budget. Can they afford to finish the basement right away? If they can afford to finish the basement, will it affect the projected move-in date? In this example, the project sponsors are also the customers and users for the product, which is the house.

- The project manager in this example would normally be the general contractor responsible for building the house. He or she needs to work with all the project stakeholders to meet their needs and expectations.

- The project team for building the house would include several construction workers, electricians, carpenters, and so on. These stakeholders would need to know exactly what work they must do and when they need to do it. They would need to know if the required materials and equipment will be at the construction site or if they are expected to provide the materials and equipment. Their work would need to be coordinated since there are many interrelated factors involved. For example, the carpenter cannot put in kitchen cabinets until the walls are completed.

- Support staff might include the buyers’ employers, the general contractor’s administrative assistant, and other people who support other stakeholders. The buyers’ employers might expect their employees to still complete their work but allow some flexibility so they can visit the building site or take phone calls related to building the house. The contractor’s administrative assistant would support the project by coordinating meetings between the buyers, the contractor, suppliers, and so on.

- Building a house requires many suppliers. The suppliers would provide the wood, windows, flooring materials, appliances, and so on. Suppliers would expect exact details on what items they need to provide, where and when to deliver those items, and so on.

- There may or may not be opponents of a project. In this example, there might be a neighbour who opposes the project because the workers are making so much noise that she cannot concentrate on her work at home, or the noise might wake her sleeping children. She might interrupt the workers to voice her complaints or even file a formal complaint. Or, the neighbourhood might have association rules concerning new home design and construction. If the homeowners did not follow these rules, they might have to halt construction due to legal issues.

As you can see from this example, there are many different stakeholders on projects, and they all have different interests. Stakeholders’ needs and expectations are important in the beginning and throughout the life of a project. Successful project managers develop good relationships with project stakeholders to understand and meet their needs and expectations.

d.) Project Life Cycle
In Contemporary Systems Analysis, 5th Edition published by Business and Educational Technologies, Marvin Gore and John Stubbe wrote that the Project Life Cycle includes the following Phases and activities:

A. Study Phase
   1. User Need
   2. Initial Investigation
   3. User Review
   4. System Performance Design
   5. Candidate Review
   6. Study Phase Report

B. Design Phase
   1. General System Review
   2. Processing Requirements Identification
3. Data Base Design
4. Control Requirements
5. Output Design
6. Input Design
7. Software Selection
8. Equipment Selection/Acquisition
9. People
11. Plans
12. Design Specifications Preparation
13. Design Phase Report Preparation

C. Development Phase
1. Implementation Planning
2. Computer Program Design
3. User Review
4. Equipment Acquisition and Installation
5. Coding and Debugging
6. Computer Program Testing
7. System Testing
9. Personnel Training
10. Changeover Plan Preparation
11. Development Phase Report Preparation
12. User Acceptance Review

D. Operation Phase
1. System Changeover
2. Routine Operation
3. System Performance Evaluation
4. System Changes/Enhancements

e.) Project Management Tools
Project management tools and techniques assist project managers and their teams in carrying out work in all nine knowledge areas. For example, some popular time-management tools and techniques include Gantt charts, project network diagrams, and critical path analysis. Table 1 lists some commonly used tools and techniques by knowledge area. You will learn more about these and other tools and techniques throughout this text.

Table 1: Common Project Management Tools and Techniques by Knowledge Area

<table>
<thead>
<tr>
<th>KNOWLEDGE AREA/CATEGORY</th>
<th>TOOLS AND TECHNIQUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integration management</td>
<td>Project selection methods, project management methodologies, management stakeholder analyses, project charters, project management plans, project management software, change requests, change control boards, project review meetings, lessons-learned reports</td>
</tr>
<tr>
<td>Scope management</td>
<td>Scope statements, work breakdown structures, statements of work, requirements analyses, scope management plans, scope verification techniques, and scope change controls</td>
</tr>
<tr>
<td>Time management</td>
<td>Gantt charts, project network diagrams, critical path analysis, crashing, fast tracking, schedule performance measurements</td>
</tr>
<tr>
<td>Cost management</td>
<td>Net present value, return on investment, payback analysis, earned value management, project portfolio management, cost estimates, cost management plans, cost baselines</td>
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<td>-----------------------------------------------------</td>
<td>--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Quality management</td>
<td>Quality metrics, checklists, quality control charts, Pareto diagrams, fishbone diagrams, maturity models, statistical methods</td>
</tr>
<tr>
<td>Human resource management</td>
<td>Motivation techniques, empathic listening, responsibility management assignment matrices, project organizational charts, resource histograms, team building exercises</td>
</tr>
<tr>
<td>Communications management</td>
<td>Communications management plans, <strong>kick-off meetings</strong>, conflict management, communications media selection, status and <strong>progress reports</strong>, virtual communications, templates, project Web sites</td>
</tr>
<tr>
<td>Risk management</td>
<td>Risk management plans, risk registers, probability/impact matrices, risk rankings</td>
</tr>
<tr>
<td>Procurement management</td>
<td>Make-or-buy analyses, contracts, requests for proposals or quotes, management source selections, supplier evaluation matrices</td>
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</tbody>
</table>

- **What these tools are used for?**

Good project management deals with three factors: time, cost and performance. Projects are successful if they are completed on time, within budget, and to performance requirements. In order to bring the many components of a large project into control there is a large toolkit of techniques, methodologies, and tools. These techniques provide the tools for managing different components involved in a project: planning and scheduling, developing a product, managing financial and capital resources, and monitoring progress. However the success of a project will always rest on the abilities of a project manager and the team members.

(i) **WORK BREAKDOWN STRUCTURE (WBS)**

This tool is related to planning and scheduling a project. Basically it is a functional decomposition of the tasks of the project. The total work of the project is broken down into the major subtasks. It starts with the end objective required and successively subdividing it into manageable components in terms of size and complexity: program, project, system, subsystem, components, tasks, subtasks, and work elements.

![Figure 3. Example of Work Breakdown Structure](image-url)
It should be product- or task-oriented and should include all the necessary effort which must be undertaken to achieve the end objective. Because it defines the work required to achieve an objective and help to show the required interfaces, a WBS is useful for complex projects. However, it has got an important drawback: it does not show the timing of activities. In order to overcome this drawback, another tool can be used.

(ii) GANTT CHARTS
In 1917, Henry Gantt developed the famous Gantt chart as a tool for scheduling work in factories. A Gantt chart is a standard format for displaying project schedule information by listing project activities and their corresponding start and finish dates in a calendar format. Managers drew Gantt charts by hand to show project tasks and schedule information, and this tool provided a standard format for planning and reviewing all the work on early military projects.

Today's project managers still use the Gantt chart as the primary tool to communicate project schedule information, but with the aid of computers, it is no longer necessary to draw the charts by hand. Figure 4 displays a Gantt chart created with Microsoft Project and Figure 5 with Microsoft Excel, the most widely used project management software today.

![Example of GANTT Chart](image)

Figure 4. Example of GANTT Chart
Basically there are two basic types of Gantt Charts: Load Charts and Project Planning Charts.

* **Load Charts:**
This type of chart is useful for manufacturing projects during peak or heavy load periods. The format of the Gantt Load Chart is very similar to the Gantt Project Planning Chart but, in this case, uses time as well as departments, machines or employees that have been scheduled.

* **Project Planning Chart**
It addresses the time of individual work elements giving a time line for each activity of a project. This type of chart is the predecessor of the following tool: PERT. As it can be seen in the figure, it is really easy to understand the graph, but in developing it you need to take into consideration certain precedence relationships between the different activities of the project. On the chart, everyone is able to see when each activity starts and finishes but there is no possibility to determine when each activity may start or if we can start a particular activity before finishing the immediate predecessor activity. Therefore, we need somehow know the precedence relationships between activities. This is the main reason for using the following tools instead of using exclusively Gantt Charts.

(iii) **PERT/CPM (Critical path Method)**
The military was the key industry behind the development of several project management techniques. Members of the U.S. Navy Polaris missile/submarine project first used network diagrams in 1958. These diagrams helped managers model the relationships among project tasks, which allowed them to create schedules that were more realistic. Figure 6 displays a network diagram created using Microsoft Project. Note that the diagram includes arrows that show which tasks are related and the sequence in which team members must perform the tasks. The concept of determining relationships among tasks is essential in helping to improve project scheduling. This concept allows you to find and monitor the critical path—the longest path through a network diagram that determines the earliest completion of a project.
Both methods show precedence relationships explicitly. Although the two methods were developed independently during the fifties, they are surprisingly similar. Both methods, PERT and CPM, use a graphic representation of a project that is called "Project Network" or "CPM diagram", and it is used to portray graphically the interrelationships of the elements of a project and to show the order in which the activities must be performed.

1. **Critical Path Method (CPM) Background**
   - Need a list of activities for some project.
   - Each activity has list of *predecessors* activities.
   - An *AOA* (activity on arc) network represents the project.
   1. Start node 1 has arcs for all activities with no predecessors.
   2. Finish node \( f \) has no successors.
   3. Start node is numbered with lower number than finish node.
   4. Each arc is labelled with an activity and an activity time \( t_{ij} \).
   5. Two nodes can be connected by at most one arc.
   6. Dummy activities are added to avoid rule 5 conflicts.

2. **CPM**
   - \( ET(i) \), the *early event time* for node \( i \), is defined by
     a) determining all immediate predecessors \( j \) for node \( i \), and
     \[
     ET(i) = \max_{all\ j} \{ ET(j) + t_{ij} \},
     \]
     b) setting
   - \( LT(i) \), the *late event time* for node \( i \) is defined by
     a) determining immediate successors \( j \) to node \( i \), and
     \[
     LT(i) = \min_{all\ j} \{ LT(j) - t_{ij} \},
     \]
     b) setting
   - \( TF(i,j) \), the *total float* time, is the allowed increase in \( t_{ij} \) without delaying project:
     \[
     TF(i,j) = LT(j) - ET(i) - t_{ij}.
     \]
   - A *critical path* is a path from start to finish through activities (*critical activities*) with \( TF(i,j) = 0 \).
• \( FF(i_j) \), the free float time, is allowed increase in starting time for \((i_j)\) without delaying project:
\[
FF(i, j) = ET(j) - ET(i) - t_{ij}
\]

• LP Formulation: let \( z_j \) be time for node \( j \) event.
\[
\text{Min } z = x_f - x_1, \text{ subject to } z_j \geq x_i + t_{ij} \forall (i, j)
\]
and all \( x_i \)urs.

• Crashing the project: re-solving the problem with additional costs, and constraints on \( t_{ij} \)s.

3. Program Evaluation and Review (PERT):

   Assume \( T_{ij} \)'s are Beta random variables and compute expected value for critical path length,
   \[
   E(T_{ij}) = (a + 4m + b)/6 \quad \text{and} \quad Var(T_{ij}) = (b - a)^2 / 36
   \]

4. CPM vs. PERT

| CPM or "Critical Path Method" | PERT or "Project Evaluation and Review Technique"
<table>
<thead>
<tr>
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<th></th>
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<tbody>
<tr>
<td>Tool to analyze project and determine duration, based on identification of &quot;critical path&quot; through an activity network.</td>
<td>Another derivative of the GANTT chart</td>
</tr>
<tr>
<td>Knowledge of the critical path can permit management of the project to change duration.</td>
<td>Multiple time estimates were used for each activity that allowed for variation in activity times</td>
</tr>
<tr>
<td>A single estimate for activity time was used that did not allow for variation in activity times</td>
<td>Activity times are assumed to be random, with assumed probability distribution (&quot;probabilistic&quot;)</td>
</tr>
<tr>
<td>Activity times are assumed to be known or predictable (&quot;deterministic&quot;)</td>
<td>Activities are represented by arrowed lines between the nodes or circles</td>
</tr>
<tr>
<td>Activities are represented as nodes or circles</td>
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</tbody>
</table>

Over time, CPM and PERT merged into one technique referred to as "CPM/PERT".

• Visually easier to see precedence relationships
• Ideal for large projects with many activities
• They consist of a network of branches and nodes.
Two types:
- **Activity-on-node (AON)** -- nodes represent activities and arrows show precedence relationships.
- **Activity-on-arrow (AOA)** -- arrows represent activities and nodes are events for points in time.

**Dummy**
inserted into the network to show a precedence relationship, but it does not represent any actual passage of time.

**Activity Slack**
Slack is computed by:

\[ S_{ij} = LS_{ij} - ES_{ij} \quad \text{or} \quad S_{ij} = LF_{ij} - EF_{ij} \]

Slack enables resources to be temporarily diverted other activities to:
• avoid delays
• compensate for an inaccurate time estimate

Most network activities are estimates
• project uniqueness means little historical basis
• subject to a lot of uncertainty
• Using probabilistic methods rather than deterministic to minimize uncertainty

**Activity Scheduling**
- **Earliest Start time (ES):** The earliest time an activity can start
- **Forward pass:** Start at the first node and move forward through the network to determine the earliest start time for an activity
- **Earliest Finish time (EF):** The earliest start time plus the activity time

\[
EF_{ij} = ES_{ij} + t_{ij}
\]

- **Latest Start time (LS):** The latest time an activity can start without delaying the completion of the project beyond the critical path time

\[
LS_{ij} = LF_{ij} - t_{ij}
\]

- **Latest Finish time (LF):** The latest time an activity can be completed and still maintain the critical path time

5. **Probabilistic Time Estimates**
PERT-type approach uses 3 time estimates for each activity
- most likely time (m)
- subjective estimate of most frequent time
- optimistic time (a)
- shortest possible time (ideally)
- pessimistic time (b)
- longest time possible if everything went wrong

**Beta distribution:**
- Estimate the mean and variance of a beta distribution of the activity times.
continuous w/ no predetermined shape
others types of distribution are no more or less accurate

Human judgment element
- Process no better than network and resource estimates
- Project teams make these subjective estimates
- Knowledgeable people must determine which events must precede others and how long activities will take.

(iv) REPRESENTING A PROJECT NETWORK
In order to represent a project network, two basic elements are used:

A cycle, called "node", represents an event. An event describes a checkpoint. It does not symbolize the performance of work, but it represents the point in time in which the event is accomplished.

An arrow, called "arc", represents an activity. The network will try to reflect all the relationships between the activities.

Two simple rules govern the construction of a project network:
A. Each activity must be represented by only one directed arc or arrow.
B. No two activities can begin and end on the same two nodes or cycles.

Another element to represent a project network is a "dummy activity". To explain it, we will consider the following example:
The temptation is to represent these relationships as:

![Diagram showing relationships between activities A, B, and C]

But then we have broken the second rule earlier mentioned. To show that activities A and B precede C, whereas activity B precedes activity D, we use a dummy activity as shown in the figure:

![Diagram showing relationships between activities A, B, C, and D]

To construct a project network, first of all, we need a list of activities showing the precedence relationships between the different activities involved, a list as the following example:
Because each activity must have a unique pair of starting and ending nodes, we must use a dummy activity to draw the first four activities, as shown in the figure. Constructing a project network is a trial-and-error process. It usually takes two or three attempts to produce a neatly constructed network. After constructing the network, the duration of each activity should be shown in parenthesis. But, what is this for? With this representation we can determine the minimum completion time for the project. We do this by starting at the originating event of the network (node 1) and determining the earliest time we can start an activity, given the activities that precede it and assuming that all the activities start as soon as possible and are completed as soon as possible. For example for the first one, it would be:

\[ T = 0 \]

\[ \begin{align*}
1 & \quad \text{EARLIEST START TIME} \\
2 & \quad \text{EARLIEST FINISH TIME}
\end{align*} \]

Where \( T=0 \) is the Earliest Start Time (ES) for activity A and \( T=1 \) is the Latest Start Time (EF) for activity A. Continuing this process results in the network:

**PROJECT NETWORK**

**PROJECT NETWORK WITH EARLIEST TIME**

Notice that in the case of activity D for example, it only starts after both precedence activities B and C are completed. If everything goes as planned, the project will take 15.5 months to complete. However, every activity needs to start as early as possible for the project to be completed in 15.5 months. We can use a similar process to determine which activities we can delay, and by how much, without increasing the completion time of the project. To calculate this, we can define the "Latest Finish Time" (LF), and the "latest Start Time" (LS) for each activity, for example:
Continuing with this process we can obtain

**PROJECT NETWORK WITH EARLIEST AND LATEST TIMES**

Now we have a project network with the earliest and the latest start and finish times, where:

Then I can calculate how much I can delay an activity. That is the "Slack Time." To determine it, we can use either or two equations:

Slack Time: \( \text{LS} - \text{ES} \)
Slack Time: \( \text{LF} - \text{EF} \)

The slack represents how long we can delay the activity without delaying the entire project. The activities that have zero slack lie on a path through the network. This path is called the "Critical Path," and the activities are called "Critical Activities." If you delay these activities, you will delay the entire project. Every project has at least one critical path, but there can be more than one. Another procedure to determine the critical path is just noticing which is the largest path through the network, in this case A-B-D-G-H-I-K-L.

**PERT/CPM: DIFFERENCES**

Both tools lead to the same end: a critical path and critical activities with slack time equal to zero. The differences between these tools come from how they treat the activity time. PERT treats activity time as a random variable whereas CPM requires a single deterministic time value for each activity. Another difference is that PERT focuses exclusively on the time variable whereas CPM includes the analysis of the Time/Cost Trade-off.

**PERT**

We have a high degree of uncertainty in regard to the completion time of many activities. It makes sense in the real world that you do not really know how long a particular activity will take, specially talking about certain activities such as research and development. In this case, we can look at the project completion time in a probabilistic fashion and for each activity we can define:

a. Optimistic time estimate: an estimate of the minimum time an activity will require.

b. Most likely time estimate: an estimate of the normal time an activity will require.

c. Pessimistic time estimate: an estimate of the maximum time an activity will require.

These three estimates are considered to be related in the form of unimodal probability distribution. m. What we need in any case is a specific duration for each activity taking into consideration these three estimates. This can be possible calculating the expected or mean activity time for each activity as
With the expected time for each activity we can determine which is the critical path. Using three assumptions, we can conclude that project completion time or critical path completion time has a normal distribution. Using this, we can determine probabilities, using completion time as a normal random variable, mean and standard deviation.
Project Management Tools

In brief:

A Gantt chart lists tasks in a project on a timeline with their interdependencies. It often also shows who is responsible for what task. It is especially useful for planning tasks in a project, and monitoring progress as the project goes on. Gantt charts emphasize time rather than task relationships.

A PERT chart, in comparison, looks more like a flow chart and concentrates on the relationships between tasks (especially their dependencies) and less on the timeline. PERT charts emphasize task relationships rather than time.

Both tools are commonly used, and they are often both used for the same project. The PERT chart is sometimes preferred over the Gantt chart because it clearly illustrates task dependencies. On the other hand, the PERT chart can be much more difficult to interpret, especially on complex projects. Frequently, project managers use both techniques.

A PERT chart is a project management tool used to schedule, organize, and coordinate tasks within a project. PERT stands for Program Evaluation Review Technique, a methodology developed by the U.S. Navy in the 1950s to manage the Polaris submarine missile program. A similar methodology, the Critical Path Method (CPM), which was developed for project management in the private sector at about the same time, has become synonymous with PERT, so that the technique is known by any variation on the names: PERT, CPM, or PERT/CPM.

A PERT chart presents a graphic illustration of a project as a network diagram consisting of numbered nodes (either circles or rectangles) representing events, or milestones in the project linked by labelled vectors (directional lines) representing tasks in the project. The direction of the arrows on the lines

* Numbered rectangles are nodes and represent events or milestones.
* Directional arrows represent dependent tasks that must be completed sequentially.
* Diverging arrow directions (e.g. 1-2 & 1-3) indicate possibly concurrent tasks.
* Dotted lines indicate dependent tasks that do not require resources.
indicates the sequence of tasks. In the diagram, for example, the tasks between nodes 1, 2, 4, 8, and 10 must be completed in sequence. These are called dependent or serial tasks.

The tasks between nodes 1 and 2, and nodes 1 and 3 are not dependent on the completion of one to start the other and can be undertaken simultaneously. These tasks are called parallel or concurrent tasks. Tasks that must be completed in sequence but that don't require resources or completion time are considered to have event dependency. These are represented by dotted lines with arrows and are called dummy activities.

For example, the dashed arrow linking nodes 6 and 9 indicates that the system files must be converted before the user test can take place, but that the resources and time required to prepare for the user test (writing the user manual and user training) are on another path. Numbers on the opposite sides of the vectors indicate the time allotted for the task.

PREVIOUS STEPS

Before attempting to use these tools, the project's information must be assembled in a certain way. I include a basic description of the preceding steps.

The project planning process consists of the following:
1. Setting the project start date
2. Setting the project completion date
3. Selecting the project methodology or project life cycle to be used
4. Determining the scope of the project in terms of the phases of the selected project methodology or project life cycle
5. Identifying or selecting the project review methods to be used
6. Identifying any predetermined interim milestone or other critical dates which must be met.
7. Listing tasks, by project phase, in the order in which they might be accomplished.
8. Estimating the personnel necessary to accomplish each task
9. Estimating the personnel available to accomplish each task
10. Determining skill level necessary to perform each task
11. Determining task dependencies
   - Which tasks can be done in parallel
   - Which tasks require the completion of other tasks before they can start
12. Project control or review points
13. Performing project cost estimation and cost-benefit analysis

Work breakdown Structures

The development of a project plan is predicated on having a clear and detailed understanding of both the tasks involved, the estimated length of time each task will take, the dependencies between those tasks and the sequence in which those tasks have to be performed. Additionally, resource availability must be determined in order to assign each task or group of tasks to the appropriate worker.

One method used to develop the list of tasks is to create what is known as a work breakdown structure.

A definition

A work breakdown structure (WBS) is a hierarchic decomposition or breakdown of a project or major activity into successive levels, in which each level is a finer breakdown of the preceding one. In final form a WBS is very similar in structure and layout to a document outline. Each item at a specific level of a WBS is numbered consecutively (e.g., 10, 10, 30, 40, 50). Each item at the next level is numbered within the number of its parent item (e.g., 10.1, 10.2, 10.3, 10.4).
The WBS may be drawn in a diagrammatic form (if automated tools are available) or in a chart resembling an outline.

The WBS begins with a single overall task representing the totality of work to be performed on the project. This becomes the name of the project plan WBS. Using a methodology or system life cycle (analysis, design and implementation) steps as a guide, the project is divided into its major steps. The first phase is project initiation; the second major phase is analysis, followed by design, construction, testing, implementation, and post-implementation follow-up. Each of these phases must be broken in their next level of detail, and each of those, into still finer levels of detail, until a manageable task size is arrived at. The first WBS level for the life cycle would be:

<table>
<thead>
<tr>
<th>WBS number</th>
<th>Task Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>Project initiation</td>
</tr>
<tr>
<td>1.1</td>
<td>Draft project plan</td>
</tr>
<tr>
<td>2.0</td>
<td>Analysis phase</td>
</tr>
<tr>
<td>2.1</td>
<td>Plan user interviews</td>
</tr>
<tr>
<td>2.2</td>
<td>Schedule users interviews</td>
</tr>
<tr>
<td>3.0</td>
<td>Examination and test</td>
</tr>
<tr>
<td>4.0</td>
<td>Design</td>
</tr>
<tr>
<td>5.0</td>
<td>Test</td>
</tr>
<tr>
<td>6.0</td>
<td>Implementation</td>
</tr>
<tr>
<td>7.0</td>
<td>Post implementation review</td>
</tr>
</tbody>
</table>

Tasks at each successively finer level of detail are numbered to reflect the task from which they were derived. Thus, the first level of tasks would be numbered 1.0, 2.0, 3.0, and so forth. Each of their subtasks would have a two part number: the first part reflecting the parent task and the second part, the subtask number itself, such as 1.1, 1.2, or 1.3.

As each of these, in turn, decomposed or broken down into its component tasks, each component receives a number comprised of it's parent number plus a unique number of its own.

A definition
A **manageable task** is one in which the expected results can be easily identified; success, failure, or completion of the task can be easily ascertained; the time to complete the task can be easily estimated; and the resource requirements of the task can be easily determined.

**Program Evaluation and Review Technique (PERT)**

Program evaluation and review technique (PERT) charts depict task, duration, and dependency information. Each chart starts with an initiation node from which the first task, or tasks, originates. If multiple tasks begin at the same time, they are all started from the node or branch, or fork out from the starting point. Each task is represented by a line which states its name or other identifier, its duration, the number of people assigned to it, and in some cases the initials of the personnel assigned. The other end of the task line is terminated by another node which identifies the start of another task, or the beginning of any slack time, that is, waiting time between tasks.

Each task is connected to its successor tasks in this manner forming a network of nodes and connecting lines. The chart is complete when all final tasks come together at the completion node. When slack time exists between the end of one task and the start of another, the usual method is to draw a broken or dotted line between the end of the first task and the start of the next dependent task.
A PERT chart may have multiple parallel or interconnecting networks of tasks. If the scheduled project has milestones, checkpoints, or review points (all of which are highly recommended in any project schedule), the PERT chart will note that all tasks up to that point terminate at the review node. It should be noted at this point that the project review, approvals, user reviews, and so forth all take time. This time should never be underestimated when drawing up the project plan. It is not unusual for a review to take 1 or 2 weeks. Obtaining management and user approvals may take even longer.

When drawing up the plan, be sure to include tasks for documentation writing, documentation editing, project report writing and editing, and report reproduction. These tasks are usually time-consuming, so don't underestimate how long it will take to complete them.

PERT charts are usually drawn on ruled paper with the horizontal axis indicating time period divisions in days, weeks, months, and so on. Although it is possible to draw a PERT chart for an entire project, the usual practice is to break the plans into smaller, more meaningful parts. This is very helpful if the chart has to be redrawn for any reason, such as skipped or incorrectly estimated tasks.

Many PERT charts terminate at the major review points, such as at the end of the analysis. Many organizations include funding reviews in the projects life cycle. Where this is the case, each chart terminates in the funding review node.

Funding reviews can affect a project in that they may either increase funding, in which case more people have to made available, or they may decrease funding, in which case fewer people may be available. Obviously more or less people will affect the length of time it takes to complete the project.

### Critical Path Method (CPM)

Critical Path Method (CPM) charts are similar to PERT charts and are sometimes known as PERT/CPM. In a CPM chart, the critical path is indicated. A critical path consists that set of dependent tasks (each dependent on the preceding one) which together take the longest time to complete. Although it is not normally done, a CPM chart can define multiple, equally critical paths. Tasks which fall on the critical path should be noted in some way, so that they may be given special attention. One way is to draw critical path tasks with a double line instead of a single line.

Tasks which fall on the critical path should receive special attention by both the project manager and the personnel assigned to them. The critical path for any given method may shift as the project progresses; this can happen when tasks are completed either behind or ahead of schedule, causing other tasks which may still be on schedule to fall on the new critical path.

### Gantt Charts

A Gantt chart is a matrix which lists on the vertical axis all the tasks to be performed. Each row contains a single task identification which usually consists of a number and name. The horizontal axis is headed by columns indicating estimated task duration, skill level needed to perform the task, and the name of the person assigned to the task, followed by one column for each period in the project's duration. Each period may be expressed in hours, days, weeks, months, and other time units. In some cases it may be necessary to label the period columns as period 1, period 2, and so on.

The graphics portion of the Gantt chart consists of a horizontal bar for each task connecting the period start and period ending columns. A set of markers is usually used to indicate estimated and actual start and end. Each bar on a separate line, and the name of each person assigned to the task is on a separate line. In many cases when this type of project plan is used, a blank row is left between tasks. When the project is under way, this row is used to indicate progress, indicated by a second bar which starts in the period column when the task is actually started and continues until the task is actually completed.
Comparison between estimated start and end and actual start and end should indicate project status on a task-by-task basis.

Variants of this method include a lower chart which shows personnel allocations on a person-by-person basis. For this section the vertical axis contains the number of people assigned to the project, and the columns indicating task duration are left blank, as is the column indicating person assigned. The graphics consists of the same bar notation as in the upper chart indicates that the person is working on a task. The value of this lower chart is evident when it shows slack time for the project personnel, that is, times when they are not actually working on any project.

<table>
<thead>
<tr>
<th><strong>Project Management Terms</strong></th>
</tr>
</thead>
</table>
| **Lead time** | Occurs when a task should theoretically wait for its predecessor to finish, but can actually start a little early. The time that the tasks overlap is lead time. For example, when replacing computers in a computer lab, you could actually start bringing in the new computers while the old ones were being packed up and moved out. The time during which packing and unpacking can happen at the same time is lead time.  
*Not to be confused with the economic "Lead time" which refers to the time between conceiving an idea and bringing it to fruition.  
Lead time, according to IT@Work, are waiting times (like lag times) that do not involve dependencies.  
MS Project's help says: "Lead time is overlap between tasks that have a dependency. For example, if a task can start when its predecessor is half finished, you can specify a finish-to-start dependency with a lead time of 50 percent for the successor task." |
| **Lag time** | Lag is the minimum amount of time that must pass between the finish of one activity and the start of its successor(s). For example, if task A is painting a wall, and dependent task B is putting up a picture on the wall, there would need to be some lag time between the end of task A and the start of task B to let the paint dry.  
Lag time is shown in a PERT chart as an arrow with a duration but no task assigned to it.  
MS Project's help says: "Lag time is a delay between tasks that have a dependency. For example, if you need a two-day delay between the finish of one task and the start of another, you can establish a finish-to-start dependency and specify two days of lag time." |
| **Float time** | A synonym for slack time. |
| **Slack time** | The amount of time a task can be extended or delayed before it impacts on the starting time of other tasks. Tasks A and B start at the same time. Task C is dependent on both tasks A and B. Task A takes 2 days and task B takes 5 days. Task A has 3 days' slack. It can run 3 days overtime before it affects the planned starting time for task C. |
| **Milestone** | An event of zero duration that marks a significant point of progress in a project. Milestones are used to see whether a project is on time or not. A milestone may be "Design is finished", "Sign contract", "Project Ends" etc. |
| **Event** | Something that happens, but that is not a task (it takes time, but does not take labour or resources). e.g. "Concrete dries" |
| **Task** | Something that needs to be done that requires some time. |
| **Dependent task** | A task that cannot begin before a previous task finishes. e.g. Task B, 'putting on shoes' is dependent on task A, 'putting on socks'. |
| **Critical path** | The series of tasks from the beginning of a project up to its end that takes the longest time. This is also the shortest time in which all tasks in the project can be completed. By definition, no task on the critical path can have any slack time. In other words, no task on the critical path can be delayed or extended without affecting the finishing date of the project. |
| **Critical task** | A task on the critical path. |
Table showing tasks, durations, resources (people responsible) and dependencies. To make a WBS accurate, you have to know the constraints of the environment. E.g. Constraint - the family's house has two showers.

**Dummy Task**

Shown by a dotted arrow on a PERT chart, it shows a dependency but no task. I know that means nothing, so see the Buffalo PERT example.

Here's another take on dummy tasks that may make sense to you.

![PERT Chart](image)

* Fig. 1: PERT Chart

- Numbered rectangles are nodes and represent events or milestones.
- Directional arrows represent dependent tasks that must be completed sequentially.
- Diverging arrow directions (e.g. 1-2 & 1-3) indicate possibly concurrent tasks
- Dotted lines indicate dependent tasks that do not require resources.

Dummy tasks are tasks that must be completed in sequence but that **don't require resources or completion time**. These are represented by dotted lines with arrows.

For example, the dashed arrow linking nodes 6 and 9 indicates that task 9 can't start until "conversion" (as well as "training" and "test system" are all finished, but nothing needs to be done by anyone between the end of conversion and the start of the next task.
PERT Introduction

A PERT chart is a project management tool used to schedule, organize, and coordinate tasks within a project. PERT stands for **Program Evaluation Review Technique**, a methodology developed by the U.S. Navy in the 1950s to manage a nuclear submarine missile program.

Below is a PERT chart drawn to show the development of a system.

![PERT Chart Image]

Yes - it was not drawn very well, but it's late and I'm tired.

Here are the questions to ponder before we discuss the answers.

- **EXAM QUESTION 1:** Which tasks are on the critical path of the PERT chart above? (1 mark)
- **EXAM QUESTION 2:** What is the slack time for tasks C, D and G? (1 mark)
- **EXAM QUESTION 3:** The person working on task C tells the project manager that he can't start work until one day after the scheduled starting date. What impact would this have on the completion date of the project? Why? (2 marks)
- **EXAM QUESTION 4:** Task A will be delayed by 2 days because some equipment has arrived late. If the project manager still wants to finish the project within the original time frame, he will need to shorten the time for one or more of the tasks. What steps can he take to reduce the number of days allocated to a task? (2 marks)
- **EXAM QUESTION 5:** The project manager decides to reduce the time needed for tasks D and F by one day each. How effective will this reduction be in achieving his aim of maintaining the original finish time for the project? (2 marks)

The CIRCLES mark the beginnings and ends of TASKS to be done in the project. Also called NODES.

The ARROWS are the tasks themselves. They are identified by letters A to I. In a real PERT chart, the actual names of tasks would be used instead of letters. The lengths of the arrows do not relate to their length in time.

The NUMBERS after the task names are the DURATIONS of the task. The time interval may be anything from picoseconds to years. Let's assume these timings are in days.

Important point to remember: the ARROWS are tasks, not the circles (nodes). When a node has two or more tasks branching from it, it means those tasks can be done concurrently (at the same time.)
When a node has incoming arrows, it means the incoming task must be completed before progress may continue to any arrows heading away from the node. e.g. Task A must be completed before tasks B or G may begin.

You need to be able to examine and interpret charts like this PERT. Let's examine it in English.

- Task A is the first task and takes 2 days. When it is done, tasks B and G can begin. If we follow the task G line, it takes 2 days to reach task H which takes 5 days. Task H leads to the final task, I. Total time for following this path is $2 + 2 + 5 + 3 = 12$ days. The path would be described as A,B,G,H,I.

- When task G began, so did task B (with another team of workers). When task B finished, after 3 days, there is another opportunity to run some tasks concurrently. So after B, tasks C and D began at the same time.
  - If we follow task C, it takes 1 day to reach task E, which leads to the final task I. Total time for this path was $2 + 3 + 1 + 4 + 3 = 13$ days.
  - If we followed task D, which takes 3 days, it leads to task F (also 3 days) before reaching the final task, I. Total time for this path is $2 + 3 + 3 + 3 + 3 = 14$ days.

- Note that tasks E, F and H must all be finished before task I can begin.

You will have noticed that there are several paths through from task A to task I. Each of these paths takes a different amount of time.

What is the shortest possible time for the project to take (without leaving any tasks out)?

14 days (the longest possible path). Yeah, it sounds odd that the shortest time is the longest path, but consider another example. You are getting ready for school. At the kitchen table, you have to have breakfast while you finish your maths homework. You have to finish both before you can leave. Breakfast takes 12 minutes. Maths takes 20 minutes. What's the shortest time you would need to leave? Twenty minutes, because both tasks must be finished. Just because one task finishes before the other, you can't leave yet. So in the chart above, the shortest project time would be 14 days.

That is the CRITICAL PATH of the project: the sequence of tasks from beginning to end that takes the longest time. No task on the critical path can take more time without affecting the end date of the project. In other words, none of the tasks on the critical path has any SLACK.

SLACK is the amount of extra time a task can take before it affects a following task. In the breakfast example above, the breakfast could take another eight minutes before it affected the leaving time, so it has eight minutes' slack.

Tasks on the critical path are called CRITICAL TASKS. No critical task can have any slack (by definition).

**EXAM QUESTION 1:** Which tasks are on the critical path of the PERT chart above? *(1 mark)*

**ANSWER:** A,B,D,F,I

**EXAM QUESTION 2:** What is the slack time for tasks C, D and G? *(1 mark)*

a. Slack time for task C: let's isolate that bit of the PERT chart.
To work out the slack time for a task, backtrack from the task to the node where the task split off from other concurrent tasks. In our case, it is the node directly before task C. Also look forward to the node where task C and the other concurrent tasks (D,F) join up again with C.

In the picture below, the beginning node is marked red and the ending node is marked green...

The top pair of tasks (C,E) is being done at the same time as the bottom pair of tasks (D,F). Together, C and E take 1 + 4 = 5 days. Together, D and F take 3 + 3 = 6 days, so tasks C,E will finish 1 day before D, F finish. Therefore, either task C or task E could take one extra day to finish without disturbing the task that comes after the green node.

That is the SLACK time for task C (it equally applies to task E; but remember the slack time is shared between them. They can't both take another day without causing delays.)

So, the slack time available to task C is ONE DAY.

b. Good. Let's try the next question: what is the slack time for task D? This is easy, when you remember that task D is on the critical path. By definition, critical tasks HAVE NO SLACK: they cannot run overtime without affecting the ending date of the project. So, the easy answer for this is THERE IS NO SLACK for task D.

c. Finally, what is the slack time for task G? Let's isolate the relevant bits of the chart again...
Once again, I have gone back from task G to the (red) node where it branches off from a concurrent task. We look ahead to where task G's path rejoins its concurrent brothers (the green node).

Tasks G,H take a total of 7 days. Meanwhile, tasks B,C,E take $3 + 1 + 4 = 8$ days and tasks B,D,F take $3 + 3 + 3 = 9$ days. So, task G could run an extra 2 days before it caused delays, since it had to wait for tasks B,D,F to finish anyway.
So the answer would be, The slack time for task G is 2 days.

**EXAM QUESTION 3:** the person working on task C tells the project manager that he can't start work until one day after the scheduled starting date. What impact would this have on the completion date of the project? Why? (2 marks)
Let's look at the whole PERT chart again...

Task C starting one day late is not significant to the ending date of the project. It would cause task E to start a day late (because task E is dependent on task C finishing first), but remember earlier we found that task C had ONE DAY OF SLACK. Therefore, if task C started a day late, it would merely use up its day of slack and no disruption would be felt by the time task E finished and the other concurrent tasks joined up to begin task I. So the answer is: task C finishing one day late would have no impact on the completion date of the project because it has one day of slack it could use.

**EXAM QUESTION 4:** Task A will be delayed by 2 days because some equipment has arrived late. If the project manager still wants to finish the project within the original time frame, he will need to shorten the time for one or more of the tasks. What steps can he take to reduce the number of days allocated to a task? (2 marks)

This is not really a PERT question at all: it is a common sense question. How can you finish a job more quickly? You could put more people to work on it, you could work more hours in a day or you could increase the efficiency of work (e.g. automating a manual task). Always remember common sense is
your most valuable tool in an exam!

**EXAM QUESTION 5:** The project manager decides to reduce the time needed for tasks D and F by one day each. How effective will this reduction be in achieving his aim of maintaining the original finish time for the project? (2 marks)

Keep in mind that tasks D and F were chosen because they were on the critical path, and the only way to affect the finishing date is to affect critical tasks. Let's modify the PERT chart to show the new timeline if the manager shortened the time needed for tasks D and F.

![PERT Chart](chart.png)

Have a think: what has changed?
YES! Reducing tasks A,B,D,F,I by a total of 2 days (to 12 days) means it is no longer the critical path! It is no longer the longest route from start to finish. In other words, it has been demoted.

What is the new critical path?
Right: Path A,B,C,E,I is now the longest at 13 days so it becomes the new critical path.

So - the project manager has reduced the old critical path from 14 to 12 days, but the new critical path still takes 13 days. So, the project will now finish ONE day earlier than originally.

Reducing both tasks D and F by one day each was unnecessary because after a 1 day reduction, the tasks were no longer critical. To reduce the overall project time further, the manager would have had to shift his attention to the new critical path, and try to reduce the times of the new critical tasks.

So, an answer could be: *Reducing tasks D and F each by one day would only shorten the project by one day since after a 2 day reduction, tasks D and F are no longer on the critical path. To further shorten the project time, the project manager would have to shorten tasks on the new critical path A,B,C,E,I.*

There are a couple of difference flavours of PERT charts: Activity on Arrow and Activity on Node

<table>
<thead>
<tr>
<th>Task</th>
<th>Predecessors Tasks (Dependencies)</th>
<th>Time (Weeks)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-</td>
<td>3</td>
</tr>
<tr>
<td>B</td>
<td>-</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>-</td>
<td>7</td>
</tr>
<tr>
<td>D</td>
<td>A</td>
<td>8</td>
</tr>
<tr>
<td>E</td>
<td>B</td>
<td>5</td>
</tr>
<tr>
<td>F</td>
<td>C</td>
<td>5</td>
</tr>
<tr>
<td>G</td>
<td>E</td>
<td>4</td>
</tr>
<tr>
<td>H</td>
<td>F</td>
<td>5</td>
</tr>
<tr>
<td>I</td>
<td>D</td>
<td>6</td>
</tr>
<tr>
<td>J</td>
<td>G - H</td>
<td>4</td>
</tr>
</tbody>
</table>
PERT Diagram using Activity On Arrow (AOA) convention

- The critical path is through activities C, F, H, J (can you prove it?)
- The expected project duration is 21 weeks (7+5+5+4)

PERT Diagram using Activity On Node (AON) convention

An as a special bonus, here’s an exercise to test your PERTness
This problem comes from the 2000 end of year IPM exam. It has been roundly criticised for its difficulty and vagueness. Let's see what we can do with it.

This Gantt chart was produced using Microsoft Project. This is a common tool for producing Gantt charts, but it is only one way of doing so; other charts may look a bit different.

Look at the time scale at the top of the chart. Notice anything odd? Yes, folks. The minor date scale (under the line beginning "16 Oct '00") shows every fourth day. Makes it much harder to read. Heaven knows why the examiners did it (I can guess: it was the only way they could get it to fit onto the printed page.)

Anyway, just remember to always check the time scale before interpreting a chart.
The major scale is in units of fortnights (the first step is 16 Oct, the next is the 30th - 14 days apart.) The
dotty lines going down the chart mark beginnings of major time units. e.g. task 8 begins on the dotty line
that marks the beginning of major unit beginning December 11 so you can easily tell it starts on Dec 11.

There are 13 tasks, listed downwards in the left column.

The blue bars graphically show the length of each task (which is also visible numerically in column 2)

The arrows from the end of one task to the beginning of another mean the second task must wait for the
first task to end before the second can begin. This is called a "dependency". The earlier task is called the
"predecessor".

Most tasks follow each other in a "lock-step" manner until you reach tasks 10 and 11 which overlap:
overlapping bars mean the tasks can be carried out concurrently (at the same time.)

Both tasks 10 and 11 are dependent on task 9. Task 11 starts on the same day as task 10 and lasts one
workday longer.

Notice however that task 10's bar is a fair bit shorter than task 11's even though they are only one
workday different: this is because task 11 spans a weekend, so its bar is actually 5 days long even
though no work is done on the weekend.

The diamonds show milestones. Milestones are significant points of progress in the project. They have
zero length: no work is actually needed for a milestone. They mark major points of project development.

The black lines in the bars of the first 3 tasks indicate how much of the task has been completed so far.
You can see tasks 1 and 2 are finished and task 3 is half-finished. This lets a project manager keep track
of where the project is actually up to.

- **Quick question 1**: When does the project start?
  **Answer**: 16 October (see where task 1's bar begins on the dotty line that starts off "16 Oct")

- **Quick question 2**: When does the project end?
  **Answer**: 21 December (the milestone is dated so you can easily tell) [actually, I believe the milestone is
  wrong - see below]

- **Quick question 3**: The exam questions below are worth a total of 5 marks: how long should you
  spend on them?
  **Answer**: For 2003 and beyond, the exam is worth 90 marks and you have 120 minutes, therefore 1.333
  minutes per mark.

Here's the chart again so you don't have to scroll so far...
Exam Question 1: What is the first milestone in this project? (1 mark)
This is an easy one: it's the first diamond you can find, which is task 7, "Sign contract" on 30/11/00

Exam Question 2: How much 'slack' time is there after commencement of the alterations, including weekends? (1 mark)
First let's find when the alterations commence: this is task 8, "Prepare site" on 11/12/00.

Now let's mentally expand the time scale (you could mark the days off with a pen). Note that while task 11's bar spans the weekend and is 5 days long in total, only 3 of the days are work days.

Let's find the critical path, starting at task 8: what is the longest possible path of tasks until the end? Tasks 8,9,11,12 add up to 1+2+3+1=7 days.

The alternate path (since tasks 10 and 11 overlap) is Tasks 8,9,10,12 adding up to 1+2+2+1=6 days.

So, there is a 1 day difference between the paths BUT the question adds "including weekends." So, if task 10 really got in trouble, and they needed all the time they could muster without affecting the finish date, they could work the weekend as well, giving them 3 slack days maximum.

Another way to approach it is to just looking at where the 2 tasks diverge (14 Dec) until the point they converge to a single path again at task 12 on 19 Dec (the same technique used in the PERT chart tutorial) The shorter path [task 10] is 2 days and the longer path [task 11] is 3 days - a one day difference. Add on the weekend as potential working time, as the question says, and the slack is 3 days.

(If you look at the original printed exam, it is VERY hard to read any detail from it and teachers have given the examiners detention for this. If there is a Gantt chart in the next exam, it must be easier to read than the one in 2000: it couldn't be worse!)
Exam Question 3: The supplier of the cable has just informed you that the cable delivered to you is faulty and the replacement cable will not arrive until the end of Monday 18 December. What impact will this have on the end date? (1 mark)

This discussion is subject to review once I work out where the big apparent problem is!

The examiners’ answer to this question was a mess. They ended up accepting 3 different answers, because many students interpreted "end date" differently.

Their official answer was: "[The end date would be delayed by] 1 day - cabling will now start on Tuesday 19th not Monday 18th but project is still able to be completed on time' or 'nothing' or 'end date will be delayed by one day' or 'end date will still be met.'"

Now call me a rebel, but I think this answer is a pile of fetid dingo’s kidneys. How can it make sense?

Here’s the end of the Gantt chart in more detail:

<table>
<thead>
<tr>
<th>Task Name</th>
<th>Duration</th>
<th>11 Dec ’00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prepare site</td>
<td>1 day</td>
<td>S M T W T F S S M T W T F</td>
</tr>
<tr>
<td>Install benches</td>
<td>2 days</td>
<td></td>
</tr>
<tr>
<td>Install power points</td>
<td>2 days</td>
<td></td>
</tr>
<tr>
<td>Install cabling</td>
<td>3 days</td>
<td></td>
</tr>
<tr>
<td>Test cabling</td>
<td>1 day</td>
<td></td>
</tr>
<tr>
<td>Room ready for hardware</td>
<td>0 days</td>
<td></td>
</tr>
</tbody>
</table>

Let’s take it step by step...

<table>
<thead>
<tr>
<th>TASK</th>
<th>STARTS</th>
<th>Length (days)</th>
<th>ENDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 - prepare site</td>
<td>11 Dec</td>
<td>1</td>
<td>11 Dec</td>
</tr>
<tr>
<td>9 - install benches</td>
<td>12 Dec</td>
<td>2</td>
<td>13 Dec</td>
</tr>
<tr>
<td>10 - powerpoints</td>
<td>14 Dec</td>
<td>2</td>
<td>15 Dec</td>
</tr>
<tr>
<td>11 - cabling</td>
<td>14 Dec also</td>
<td>3</td>
<td>18 Dec (incl. weekend)</td>
</tr>
<tr>
<td>12 - test cabling</td>
<td>19 Dec</td>
<td>1</td>
<td>19 Dec</td>
</tr>
<tr>
<td>13 - room ready milestone</td>
<td>20 Dec</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

1. According to my calculations, the Gantt chart’s final milestone is wrong. It is marked 21 Dec, but it should be on the 20th.

2. Since the cabling should have started on the 14th of December (not the 18th as the examiners’ answer claims), a delivery delay causing installation to start on December 19th would make the end of the Gantt chart look like this:
Therefore, the finishing date of the project would have changed from end end of Tuesday 19 Dec to the
end of Friday 22 Dec - a total of THREE days' delay to the 'end date', if you interpret 'end date' as the
original planned finishing day: December 19.

If you interpreted 'end date' as the date the room had to be finished by (Dec 22 in the case study), there is
no change to the 'end date'.

This fuzzy definition of 'end date' was the reason the examiners ended up with a bag of acceptable
answers to the question.

Anyway, the examiners accepted 3 different answers (not including mine above.) I still think mine is valid,
if interpreting 'end date' as 'original expected finishing date' (Dec 19).

- **Exam Question 4:** What options would a project manager have if the cable was arriving on 21
December and the due date still had to be met? (2)

This answer has **nothing to do with the Gantt chart**: it is a common question to test common sense. It is
really asking, "How could a task be hurried up?" Expect this type of question and remember that it is not
related to the Gantt chart you are given for the other questions.

The sorts of things you can do are:
- work 24 hours a day;
- bring in more workers;
- reduce the time taken on other tasks (such as cable testing);
- improve work efficiency (e.g. use an electric screwdriver to tighten screws rather than using a
manual screwdriver or replace inexperienced workers with skilled and experienced staff who
would work faster.)

Methods that 'cut corners' and affect the quality of the work are not recommended! Suggestion 3 above
(cutting down on testing time) is perilously close to 'cutting corners.'
PROGRAM EVALUATION AND REVIEW TECHNIQUE (PERT) AND CRITICAL PATH METHOD (CPM)

Program Evaluation and Review Technique (PERT) and Critical Path Method (CPM) are tools widely used in project scheduling. Both are based on network diagrams applicable for both the planning and control aspects of production. Visual display of the network enhances the communication and highlights the interdependency of the various activities required for project completion. Perhaps the greatest contribution of these tools is the identification of sequentially time-critical activities that require the closest monitoring.

BACKGROUND
In the early 1900s the Gantt chart was widely hailed as the reason that ships were built in record time. Developed by an engineer named Henry Gantt, this horizontal bar chart shows the scheduled times for individual jobs to be accomplished by specific resources. However, this tool is static in nature, and requires frequent manual updating, especially when activities are sequentially dependent.

In Figure 1, the Gantt chart shows the prospective times for five activities in a project, but does not show an underlying dependency of Activity D on the completion of Activity B.

In the 1950s, two groups independently developed what has become known as the PERT/CPM method of project scheduling. Each of these techniques improved on the Gantt chart by building into the tool the explicit sequencing of activities.

PERT was developed by the U.S. Navy, the Lockheed Corporation, and the consulting firm of Booz, Allen and Hamilton to facilitate the Polaris missile project. As time was a primary issue, this technique used statistical techniques to assess the probability of finishing the project within a given period of time.

By contrast, CPM was created in the environment of industrial projects, where costs were a major factor. In addition to the identification of the time-critical path of activities, representatives from the Du Pont Company and Sperry-Rand Corporation also developed a time-cost tradeoff analysis mechanism called crashing.

These two tools differ in the network diagram display. PERT historically uses the activity-on-arrow (AOA) convention, while CPM uses activity-on-node (AON). For most purposes, these two conventions are interchangeable; however some proprietary software requires the logic of a specific convention. Both forms of network diagrams use arrows (lines implying direction) and nodes (circles or rectangles) to define the set of project activities or tasks. The flow of logic is from left to right. To simplify the diagram, letters are frequently used to represent individual activities. Figures 2 and 3 illustrate the differences for the same simple project.

Figure 2 illustrates the AOA convention, in which arrows depict activity requiring time and resources. The node represents an event, which requires neither time nor resources; this event is actually recognition that prior tasks are completed and the following tasks can begin. While the length of the arrow is not necessarily related to the duration of the task, there may be a tendency on the part of the analyst to sketch longer arrows for longer activities. To maintain the integrity of the network, there may be need for a dummy activity, as it is not acceptable to have two tasks that share the same beginning and ending nodes.

In Figure 3, the AON uses nodes to represent activities. The arrows have no implication of time, used only to indicate sequential flow. Since the AOA convention requires the use of dummy activities, the simpler AON convention will be used here to illustrate an example.
USING CPM TO SCHEDULE AND CONTROL A PROJECT

Scheduling is an important part of the planning of any project. However, it is first necessary to develop a list of all the activities required, as listed in the work breakdown structure. Activities require both time and the use of resources. Typically, the list of activities is compiled with duration estimates and immediate predecessors.
To illustrate the use of CPM, we can imagine a simple cookie-baking project: the recipe provides the complete statement of work, from which the work breakdown structure can be developed. The resources available for this project are two cooks and one oven with limited capacity; the raw materials are the ingredients to be used in preparing the cookie dough. As listed in Table 1, the activities take a total of 80 minutes of resource time. Because some activities can run parallel, the cooks should complete the project in less than 80 minutes.

Table 1 displays some of the planning that will save time in the project. For example, once the oven is turned on, it heats itself, freeing the cooks to perform other activities. After the dough is mixed, both batches of cookies can be shaped; the shaping of the second batch does not have to wait until the first batch is complete. If both cooks are available, they can divide the dough in half and each cook can shape one batch in the same four-minute period. However, if the second cook is not available at this time, the project is not delayed because shaping of the second batch need not be completed until the first batch exits the oven.

Some expertise is required in the planning stage, as inexperienced cooks may not recognize the independence of the oven in heating or the divisibility of the dough for shaping. The concept of concurrent engineering makes the planning stage even more important, as enhanced expertise is needed to address which stages of the project can overlap, and how far this overlap can extend.

After beginning the project at 8:00 A.M., the first batch of dough is ready to go into the oven at 8:14, but the project cannot proceed until the oven is fully heated—at 8:15. The cooks actually have a one-minute cushion, called slack time. If measuring, mixing, or shaping actually take one additional minute, this will not delay the completion time of the overall project.

<table>
<thead>
<tr>
<th>Description of Activity</th>
<th>Duration (minutes)</th>
<th>Immediate Predecessor(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Preheat oven</td>
<td>15 minutes</td>
<td>—</td>
</tr>
<tr>
<td>B. Assemble, measure ingredients</td>
<td>8 minutes</td>
<td>—</td>
</tr>
<tr>
<td>C. Mix dough</td>
<td>2 minutes</td>
<td>B</td>
</tr>
<tr>
<td>D. Shape first batch</td>
<td>4 minutes</td>
<td>C</td>
</tr>
<tr>
<td>E. Bake first batch</td>
<td>12 minutes</td>
<td>A, D</td>
</tr>
<tr>
<td>F. Cool first batch</td>
<td>10 minutes</td>
<td>E</td>
</tr>
<tr>
<td>G. Shape second batch</td>
<td>4 minutes</td>
<td>C</td>
</tr>
<tr>
<td>H. Bake second batch</td>
<td>12 minutes</td>
<td>E, G</td>
</tr>
<tr>
<td>I. Cool second batch</td>
<td>10 minutes</td>
<td>H</td>
</tr>
<tr>
<td>J. Store cookies</td>
<td>3 minutes</td>
<td>F, I</td>
</tr>
<tr>
<td>Total time</td>
<td>80 minutes</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4 illustrates the network diagram associated with the cookie-baking project. The set of paths through the system traces every possible route from each beginning activity to each ending activity. In this simple project, one can explicitly define all the paths through the system in minutes as follows:

\[
\begin{align*}
A-E-F-J &= 15 + 12 + 10 + 3 = 40 \\
A-E-H-I-J &= 15 + 12 + 12 + 10 + 3 = 52 \\
B-C-D-E-F-J &= 8 + 2 + 4 + 12 + 10 + 3 = 39 \\
B-C-D-E-H-I-J &= 8 + 2 + 4 + 12 + 12 + 10 + 3 = 51 \\
B-C-G-H-I-J &= 8 + 2 + 4 + 12 + 10 + 3 = 39
\end{align*}
\]

The critical path is the longest path through the system, defining the minimum completion time for the overall project. The critical path in this project is A-E-H-I-J, determining that the project can be completed in 52 minutes (less than the 80-minute total of resource-usage time). These five activities must be done in sequence, and there is apparently no way to shorten these times. Note that this critical path is not dependent on the number of activities, but is rather dependent on the total time for a specific sequence of activities.

The managerial importance of this critical path is that any delay to the activities on this path will delay the project completion time, currently anticipated as 8:52 A.M. It is important to monitor this critical set of activities to prevent the missed due-date of the project. If the oven takes 16 minutes to heat (instead of the predicted 15 minutes), the project manager needs to anticipate how to get the project back on schedule. One suggestion is to bring in a fan (another resource) to speed the cooling process of the second batch of cookies; another is to split the storage process into first- and second-batch components. Other paths tend to require less monitoring, as these sets of activities have slack, or a cushion, in which activities may be accelerated or delayed without penalty. Total slack for a given path is defined as the difference in the critical path time and the time for the given path. For example, the total slack for B-C-G-H-I-J is 13 minutes (52–39 minutes). And the slack for B-C-D-E-H-I-J is only one minute (52–51), making this path near critical. Since these paths share some of the critical path activities, it is obvious that the manager should look at the slack available to individual activities.

Table 2 illustrates the calculation of slack for individual activities. For projects more complex than the simplistic cookie project, this is the method used to identify the critical path, as those activities with zero slack time are critical path activities. The determination of early-start and early-finish times use a forward pass through the system to investigate how early in the project each activity could start and end, given the dependency on other activities.

The late-time calculations use the finish time from the forward pass (8:52 A.M.) and employ a backward pass to determine at what time each activity must start to provide each subsequent activity with sufficient time to stay on track.
Slack for the individual activities is calculated by taking the difference between the late-start and early-start times (or, alternatively, between the late-finish and early-finish times) for each activity. If the difference is zero, then there is no slack; the activity is totally defined as to its time-position in the project and must therefore be a critical path activity. For other activities, the slack defines the flexibility in start times, but only assuming that no other activity on the path is delayed.

CPM was designed to address time-cost trade-offs, such as the use of the fan to speed the cooling process. Such crashing of a project requires that the project manager perform contingency planning early in the project to identify potential problems and solutions and the costs associated with employing extra resources. Cost-benefit analysis should be used to compare the missed due-date penalty, the availability and cost of the fan, and the effect of the fan on the required quality of the cookies.

This project ends with the successful delivery of the cookies to storage, which brings two questions to mind: First, should the oven be turned off? The answer to this depends on the scheduling of the oven resource at the end of this project. It might be impractical to cool the oven at this point if a following project is depending on the heating process to have been maintained. Second, who cleans up the kitchen? Project due dates are often frustrated by failure to take the closeout stages into account.

### Table 2 Calculation of Slack Time

<table>
<thead>
<tr>
<th>Activity</th>
<th>Early Start</th>
<th>Early Finish</th>
<th>Late Start</th>
<th>Late Finish</th>
<th>Slack</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8:00</td>
<td>8:15</td>
<td>8:00</td>
<td>8:15</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>8:00</td>
<td>8:08</td>
<td>8:01</td>
<td>8:09</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>8:08</td>
<td>8:10</td>
<td>8:09</td>
<td>8:11</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>8:10</td>
<td>8:14</td>
<td>8:11</td>
<td>8:15</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>8:15</td>
<td>8:27</td>
<td>8:15</td>
<td>8:27</td>
<td>0</td>
</tr>
<tr>
<td>F</td>
<td>8:27</td>
<td>8:37</td>
<td>8:39</td>
<td>8:49</td>
<td>12</td>
</tr>
<tr>
<td>G</td>
<td>8:10</td>
<td>8:14</td>
<td>8:23</td>
<td>8:27</td>
<td>13</td>
</tr>
<tr>
<td>H</td>
<td>8:27</td>
<td>8:39</td>
<td>8:27</td>
<td>8:39</td>
<td>0</td>
</tr>
<tr>
<td>I</td>
<td>8:39</td>
<td>8:49</td>
<td>8:39</td>
<td>8:49</td>
<td>0</td>
</tr>
<tr>
<td>J</td>
<td>8:49</td>
<td>8:52</td>
<td>8:49</td>
<td>8:52</td>
<td>0</td>
</tr>
</tbody>
</table>

USING PERT TO SCHEDULE AND CONTROL A PROJECT

In repetitive projects, or in projects employing well-known processes, the duration of a given activity may be estimated with relative confidence. In less familiar territory, however, it may be more appropriate to forecast a range of possible times for activity duration. Using the same cookie-baking project example, Figure 4 still accurately represents the sequencing of activities.

Table 3 illustrates the project with three time estimates for each activity. While $m$ represents the most likely time for the activity, $a$ suggests the optimistic estimate and $b$ is the pessimistic estimate. The estimated time and or standard deviation for each activity ($E$) are calculated from the formula for the flexible beta distribution. With a reasonably large number of activities, summing the means tends to approximate a normal distribution, and statistical estimates of probability can be applied.

The mean is calculated as $[(a + 4m + b) + 6]$, an average heavily weighted toward the most likely time, $m$. The standard deviation for an activity is $[(b - a) + 6]$, or one-sixth of the range. Managers with a basic understanding of statistics may relate this to the concept of the standard deviation in the normal distribution. Since ±3 standard deviations comprise almost the entire area under the normal curve, then there is an intuitive comparison between a beta standard deviation and the normal standard deviation. Using these new estimates for activity duration, the activity paths through the system have not changed, but the estimates of total time ($T$) are as follows:
There are two factors that should be considered coincidental to the comparison of PERT and CPM in the example. First, there are two critical paths of \( T = 53 \) minutes each in the PERT analysis. Second, all the other paths have the same duration of \( T = 40.66 \) minutes. These concepts are neither more nor less likely to happen under PERT as opposed to CPM; they are strictly a function of the numbers in the estimates. However, the serendipity of two critical paths allows us to address the issue of which would be considered the more important of the two.

In Table 4, each of the critical paths is considered. Relevant to this analysis is the sum of the variances on the critical path; note that summing variances is mathematically valid, while summing standard deviations is not. Path A-E-H-I-J has a total variance of 5.78 minutes, while path B-C-D-E-H-I-J has a variance of 6.78. Thus, path B-C-D-E-H-I-J, with the larger variance, is considered the riskier of the two paths and should be the primary concern of the project manager. We assign the entire project a variance of 6.78 minutes, and the standard deviation (the square root of the project variance) is 2.60 minutes.

<table>
<thead>
<tr>
<th>Table 3. List of Project Activities (PERT)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Description of Activity</strong></td>
</tr>
<tr>
<td>A. Preheat oven</td>
</tr>
<tr>
<td>B. Assemble, measure ingredients</td>
</tr>
<tr>
<td>C. Mix dough</td>
</tr>
<tr>
<td>D. Shape first batch</td>
</tr>
<tr>
<td>E. Bake first batch</td>
</tr>
<tr>
<td>F. Cool first batch</td>
</tr>
<tr>
<td>G. Shape second batch</td>
</tr>
<tr>
<td>H. Bake second batch</td>
</tr>
<tr>
<td>I. Cool second batch</td>
</tr>
<tr>
<td>J. Store cookies</td>
</tr>
<tr>
<td><strong>Total times</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4 Variability of Project Activities (PERT)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Path = A–E–H–I–J. Description of Activity</strong></td>
</tr>
<tr>
<td>A. Preheat oven</td>
</tr>
<tr>
<td>E. Bake first batch</td>
</tr>
<tr>
<td>H. Bake second batch</td>
</tr>
<tr>
<td>I. Cool second batch</td>
</tr>
<tr>
<td><strong>Total variance</strong></td>
</tr>
<tr>
<td><strong>Standard deviation</strong></td>
</tr>
</tbody>
</table>
Path = B–C–D–E–H–I–J. Description of Activity | Duration (minutes)
---|---
B. Assemble, measure ingredients  | 6 8 12 8.33 1 1
C. Mix dough  | 2 2 2 2.00 0 0
D. Shape first batch  | 3 4 9 4.67 1 1
E. Bake first batch  | 10 12 16 12.33 1 1
H. Bake second batch  | 10 12 16 12.33 1 1
I. Cool second batch  | 5 10 11 9.33 1 1
J. Store cookies  | 2 3 10 4.00 1.78 1.33

Total variance  | 6.78
Standard deviation  | 2.60

Armed with this project standard deviation, the next step is to estimate the probability of finishing the project within a defined period. Applying the critical path time of 53 minutes to the normal distribution, the probability of finishing in exactly $T = 53$ minutes is 50/50. The relevant formula for calculating the number of standard normal distributions is as follows:

$$ Z = \left( \frac{C - T}{S} \right) $$

where $T =$ total time of the critical path ($T = 53$)

$S =$ standard deviation of the project ($S = 2.60$)

$C = $ arbitrary time for end of project

If $C = 9:00$ a.m., then $Z = \left( \frac{9:00 - 8:53}{2.60} \right) = 7 ÷ 2.60 = 2.69$ standard normal deviations. Referring to a cumulative standard normal table, we find that $Z = 0.99632$, or a 99.632 percent chance of finishing by 9:00 A.M.

If $C = 8:50$ A.M., then $Z = \left( \frac{8:50 - 8:53}{2.60} \right) = -3 ÷ 2.60 = -1.15$. In this case, we use $(1 - \text{table value})$ for the probability $= 1 - 0.87493 = 0.1251$, or a 12.51 percent chance of finishing 3 minutes earlier than predicted.

From a managerial viewpoint, it should be reiterated that there is only a 50/50 chance of completing the project within the sum of the activity-time estimates on the critical path ($T$). This perspective is not emphasized in the CPM analysis, but is likely relevant in that context also. Adding a buffer to the promised due date (where $C > T$) enhances the probability that the project will be completed as promised. There may be competitive advantages to bidding a project on the basis of a nearer-term completion date (where $C < T$), but managers can assess the risks involved using PERT analysis. In the cookie example, there may be a promised delivery time riding on this project estimate, or the resources (cooks and oven) may be promised to other projects. By using PERT, managers can allocate the resources on a more informed basis.

Both PERT and CPM rely heavily on time estimates, as derived from local experts, to determine the overall project time. While the estimating process may intimidate local managers, this may suffice to produce an estimate that becomes a *fait accompli*, as managers strive to meet the goal rather than explain why they failed to do so.

These two project management tools, frequently used together, can assist the project manager in establishing contract dates for project completion, in estimating the risks and costs of contingencies, and in monitoring project progress. Many commercial software packages exist to support the project manager in tracking both costs and time incurred to date through-out the project duration.
Critical Path Method (CPM)

The Critical Path Method (or CPM) is the method by which the critical path through a project schedule is calculated. It helps answer the question ‘How long is this project going to take?’

The critical path is the sequence of tasks which have zero float. Thus, if any task on the critical path finishes late, then the whole project will also finish late. There is always at least one critical path.

CPM is a fairly involved approach with many calculations. Fortunately, many Project Management software programs, such as Microsoft Project, do this work for you.

Activity Network

One of the 'second seven tools', this is a classic project planning tool for piecing together a multi-task complex set of interdependent activities. It allows you to calculate what task starts when, what float there is in the project, etc.

Also called an 'Activity-on-Node Diagram', the alternative (but potentially more troublesome) method of doing this is with an 'Activity-on-Arrow Diagram', or simply 'Arrow Diagram'.

Tasks are shown as boxes
Arrows show dependencies. Task 3 cannot begin until both Task 1 and Task 2 have been completed.

Task 4 is a predecessor to Task 5 and a successor to Task 2
There is both a start and end point to the project.
Float

*Float, or slack time,* is the amount of time that a task in a project schedule can be delayed without affecting the completion time of the overall project.

*Total float* is the total amount of float in a project schedule (i.e. the sum of the float for all tasks) and represents the total 'wasted time'.

*Load balancing* is the shuffling of tasks to try and minimize the total float.

Critical Path

The *critical path* through a project schedule is the sequence of tasks which have zero float. Thus, if any task on the critical path finishes late, then the whole project will also finish late. There is always at least one critical path.

Float, or slack time, is the amount of time that a task can be delayed without affecting the completion time of the overall project.

The critical path is calculated with is the *Critical Path Method* (or *CPM*). Once the start date for the overall project is known, this will give the earliest and latest start dates for each task.
Programmed Evaluation and Review Technique (PERT)

The PERT Chart is another name for the Activity Network. PERT is the older name, and is believed to have originated in the US Navy in the 1950s.

PERT stands for *Programmed Evaluation and Review Technique*.

PERT Charts are often used in combination with the Critical Path Method (CPM).
### PROJECT PLANNING AND SCHEDULING

**Introduction**

Projects - whether they involve the design of a complex water reclamation plant or the construction of a simple commercial building - need to be undertaken in an organized manner to be completed successfully. One must carefully consider what activities are required to conduct a project and how those activities are related. In the construction of a building, for example, excavations will be required, footings must be constructed, and foundations walls are needed. These activities are sequential, i.e., excavation must precede footing placement and walls cannot be built until footings are ready. The same job could include interior painting, exterior finishing, and landscaping. These items are, for the most part, independent of each other and could possibly be undertaken concurrently and, thus, may be considered non-sequential.

Project planning can be defined as identifying what activities are required to complete a project and determining the sequence in which those activities must or can be accomplished. Project scheduling involves determining how long individual activities - sometimes called tasks - will take and preparing an overall timetable. Project planning and scheduling, carried out in a systematic manner, can contribute to the success of planning, design, or construction operations as much as can the technical abilities of project personnel. Organized planning and scheduling is certainly better than a "seat of the pants" approach used so often.

Project planning and scheduling can be instrumental with many aspects of a project including the following.

- Preparing an overall schedule.
- Determining if projects can be completed in allotted times.
- Identifying activities critical to overall completion time.
- Rescheduling activities to accomplish jobs in less time.
- Allocating resources such as personnel and equipment.

This chapter introduces three planning and scheduling techniques - bar charts, critical path method, and time scale diagramming - and discusses the advantages and disadvantages of each in accomplishing the above.

**Bar Charts**

Bar charts, sometimes known as Gantt charts for Henry Gantt who developed the concept in the early twentieth century, are a graphical representation of a project's individual activity durations. As an example, Figure 5-1 lists the major items associated with construction of a light building and presents a bar chart of how those items might be scheduled. The bar chart indicates when the various jobs are to take place and is the type of schedule familiar to most persons with non-technical backgrounds.

While the bar chart in Figure 5-1 is easy to understand, it has several limitations. A major disadvantage is that there is no indication of how the various activities relate. For example, Activity H, Site Utilities, is scheduled to occur before Activity G, Roof. Is that really necessary, though? The bar chart does not answer the question. Similarly, Activity N, Landscaping, is planned to begin following the completion of Activity J, Interior Partitions. Is there any connection between the two? Again, the bar chart fails to provide the answer. Also lacking is any information as to how much slack time exists for each activity. Surely every activity in a project need not be completed right on schedule, but the bar chart does not provide data on slack time.

**Critical Path Method**

The limitations of the bar chart can be overcome with the use of techniques generally known as network or arrow diagramming. The Critical Path Method (CPM) is probably the most widely used arrow
diagramming technique. Another is Program Evaluation and Review Technique (PERT), developed to include risk analysis and probability evaluation.

With CPM, interrelationships between activities can be shown by representing each one with an arrow and connecting the arrows in the order in which the activities must be undertaken. Figure 5-2 presents information and arrow diagrams for a common operation - getting from bed to work each day. Figure 5-2(b) indicates the plan of activities. It shows, for example, that Activity B follows Activity A and that Activities C and D both follow Activity B and can occur concurrently. The flow of the operation can clearly be seen.

Note should be taken of several items in Figure 5-2(b). One, each activity starts and ends with a circle having a number inside. The circle is known as a node and the number is the node identification. Activity A could now be known as ACT 1,2 and Activity F as ACT 6,7. (Although shown here, node circles and numbers are not always used.) Two, while branching within the arrow diagram is legitimate, there are no branched beginnings or ends. All diagrams start at one node and finish at one node. Three, dummy arrows are sometimes used for convenience in drawing the figures. In Figure 5-2(b) ACT 4,6 is a dummy arrow used to connect the end of Activity C to the beginnings of Activities F and G. Dummy arrows have no duration and are used simply to indicate how activities are related.

**CPM Start and Finish Calculations**

Figure 5-2(b) presents a plan of the activities involved in getting to work. Knowing how long each job takes, a schedule of the activities can be made. Figures 5-2(c) and 5-2(d) present start and finish time calculations for the operation shown in Figure 5-2(b). Four different times associated with each activity are calculated. Earliest Start, ES, is the earliest time any job can be started from a node considering the schedule of activities which precede or terminate at that node. Earliest Finish, EF, is the earliest start time of the activity plus the duration of the activity. Latest Finish, LF, is the latest an activity can finish at a node and not delay activities which follow or commence at that node. Latest Start, LS, is the latest finish time of an activity minus the duration of the activity.

Figure 5-2(c) shows the calculations for earliest starts and earliest finishes. ACT 1,2 starts at time zero and has a duration (DUR 1,2) of 5 minutes. Its earliest start (ES 1,2) is 0 and its earliest finish (EF 1,2) is 5. Since ACT 2,3 is preceded by only ACT 1,2, EF 1,2 becomes ES 2,3. Since DUR 2,3 is 3 minutes, EF 2,3 is 8. All that is being done is the adding of durations to earliest start times, for example:

\[
\begin{align*}
EF \ 2,3 &= ES \ 2,3 + DUR \ 2,3 \\
EF \ 2,3 &= 5 \ min + 3 \ min \\
EF \ 2,3 &= 8 \ min
\end{align*}
\]

Before continuing with the calculations it is important to note a convention concerning time. ES 1,2 is the end of 0 minutes while EF 2,3 is the end of 8 minutes. Convention dictates using "end of" times. This may not seem like much when working in minutes, but when the time unit is days or weeks it is an important distinction. Thus an activity in a construction project having an ES of 19 days actually starts following the end of the 19th day (the beginning of the 20th).

Continuing with Figure 5-2(c), the plan of activities splits or branches following Node 3. EF 2,3 (end of 8 minutes) becomes ES 3,4 and ES 3,5. The process of adding durations to earliest start times continues as described above. Since ACT 4,6 is a dummy activity having no duration, ES 4,6 = EF 4,6. At Node 6 two activities (ACT 4,6 and ACT 5,6) terminate, and they have different EF's (20 minutes and 22 minutes, respectively). Since the activities which commence at Node 6 (ACT 6,7 and ACT 6,8) can not proceed until both ACT 4,6 and ACT 5,6 are complete, the greater EF (EF 5,6 = 22 minutes) becomes the earliest start for both ACT 6,7 and ACT 6,8.

Whenever two or more activities terminate at a node, the greatest EF of the terminating activities becomes the ES of the following activities. Thus, ES 8,9 is 37 minutes. Continuing, EF 8,9 is 57 minutes, the earliest finish time of the entire operation.
Figure 5-2(d) shows the calculations for latest finish and latest start times. Start by stating that EF 8,9 (57 minutes) = LF 8,9 and then subtracting DUR 8,9 (20 minutes) to get LS 8,9 (37 minutes). Two activities, ACT 6,8 and ACT 7,8, terminate at Node 8 and thus LS 8,9 becomes LF 6,8 and LF 7,8. The subtraction process continues and results in two different late start times at Node 6 (LS 6,7 = 22 minutes and LS 6,8 = 27 minutes). Since the activities which commence at Node 6 (ACT 4,6 and ACT 5,6) can not start until both ACT 4,6 and ACT 5,6 are complete, the lesser LS (LS 6,7 = 22 minutes) becomes the LF for both ACT 4,6 and ACT 5,6.

Whenever two or more activities commence at a node, the least LS of the commencing activities becomes the LF of the preceding activities. Thus LF 4,6 = LF 5,6 = 22 minutes. Note that the same situation occurs at Node 3; LF 2,3 is the lesser of LS 3,4 and LS 3,5. Continuing the process to the beginning of the project yields LS 1,2 = 0 minutes which is reasonable since the operation can begin only at time zero.

**CPM Critical Path/Float**
In addition to showing how activities are related, CPM also has the advantage over bar charts in identifying which activities are critical to the final completion time (EF/LF) of a project. CPM can also determine which activities have float or slack time and can calculate the amount of that time. With this knowledge, critical activities can be given priority attention and activities with float can be scheduled at their most convenient time (within certain limitations).

**Critical Path**
The critical path of a project is the sequence of activities for which the ES's = LS's (or EF's = LF's). Thus in Figure 5-2, the critical path consists of Activities A, B, D, E, F, and H. Note that a dummy arrow (ACT 7,8) is also on the critical path. If there is any slip in one or more of these activities, the overall operation will take longer to complete. By the same token, if one or more of these activities can be reduced in time, the overall operation can be completed more quickly.

In Figure 5-2, Activities C and G (as well as dummy ACT 4,6) are non-critical. A certain amount of slack is available in these activities without affecting overall completion time. For example, Activity G could be increased to 14 minutes without an effect on the critical path or the overall completion time. If increased to 15 minutes, a second critical path would be created. Activities G and F, on two different paths, would both be critical. If increased to 16 or more minutes, Activity G would increase completion time and Activity F would no longer be on a critical path. As shown is Figure 5-2, Activity G (and/or Activity C) could also be reduced in duration, but this would have no impact on final completion as these items are not on the critical path.

**Float**
Float is a measure of how much slack exists for a given activity. Activities C and G in Figure 5-2 have float because the duration of each could be increased somewhat without affecting final completion. Activities on the critical path have no float and, thus, another way of identifying critical path items is to determine which ones have zero float.

- **Total Float**
  Total float, TF, is the time available for doing the activity minus the time required to do the activity. Thus,
  
  \[
  TF \text{ ACT} = LF \text{ ACT} - ES \text{ ACT} - DUR \text{ ACT}
  \]
  
  For example, in Figure 5-2,
  
  TF 3,4 = LF 3,4 - ES 3,4 - DUR 3,4
  TF 3,4 = 22 min - 8 min - 12 min
  TF 3,4 = 2 min
• **Free Float**

Free float, FF, is the delay possible in an activity if all preceding activities start as early as possible while having all subsequent activities start as early as possible. Thus,

\[
FF\ ACT = ES\ following\ ACT's - ES\ ACT - DUR\ ACT
\]

(Note that ES following ACT's refers to real activities, not dummy ones.)

For example, in Figure 5-2,

\[
FF\ 3,4 = ES\ 6,7 - ES\ 3,4 - DUR\ 3,4 \\
FF\ 3,4 = 22\ min - 8\ min - 12\ min \\
FF\ 3,4 = 2\ min
\]

The effect of calculating free float is to place available float on the last activity in the chain in which the float occurs - providing a safety factor of sorts.

• **Independent Float**

Independent float, IF, is the delay possible in an activity if all previous activities are finished as late as possible and all following activities start as early as possible. Thus,

\[
IF\ ACT = ES\ following\ ACT's - LF\ preceding\ ACT's - DUR\ ACT
\]

(Note that ES following ACT's and LF preceding ACT's both refer to real activities not dummy ones.)

For example, in Figure 5-2,

\[
IF\ 3,4 = ES\ 6,7 - LF\ 2,3 - DUR\ 3,4 \\
IF\ 3,4 = 22\ min - 8\ min - 12\ min \\
IF\ 3,4 = 2\ min
\]

The effect of calculating independent float is to identify float available on one and only one activity. If an IF calculation results in a negative number, the IF is taken as zero.

In these examples, the three floats of ACT 3,4 are the same. As will be seen later, this is not always the case.

**Time Scale Diagramming**

Despite the previously discussed benefits of CPM, and its adaptability to computer calculations, there is one major drawback to CPM - the arrow diagrams developed have no time reference. This is a particular disadvantage when dealing with non-technical persons who lack planning and scheduling expertise and at best probably understand bar charts. There is, fortunately, a method which combines the benefits of bar charts and CPM.

Time scale diagramming presents arrow diagrams in which the length of an arrow is indicative of the time required for that particular activity. This is in contrast to CPM networks in which arrow lengths have no significance. Figure 5-3 presents a list of activities, associated durations, and sequencing information for a theoretical operation and a time scale arrow diagram for the project. Note that Activity A, which has a duration of 13 time units, is represented by an arrow having a horizontal length of 13 units.

The three categories of float previously discussed show up graphically on a time scale arrow diagram. Independent float, IF, shows up as a dashed arrow following an activity that is the sole activity on a branch off another path. This is illustrated by Activity G which is the only operation on that branch off the critical path. IF G = 2 time units as shown by the dashed arrow following Activity G and these 2 units are available to this activity only.

Free float, FF, shows up as a dashed arrow following the last activity on a branch off the critical path having two or more activities. For example, Activity C is followed by a dashed arrow having a length of 3 units; thus FF C = 3 time units. Further inspection of the figure will reveal that this slack of 3 units could also be taken on Activity B or shared between Activities B and C. By definition, however, free float shows
up on the last activity in the chain, and, thus, is allocated to Activity C. Investigation of Activity L shows
FF_L = 4 time units, the length of the dashed arrow immediately following this activity. IF_L = 0, however,
since the free float of 4 days could be used by Activities A, B, or C.

Total float, TF, the least restrictive of the three types of float, is the sum of the lengths of all the dashed
arrows following that activity. Thus Activity B has a total float of 7 time units, the sum of the 3 units in the
dashed arrow following Activity C and the 4 units following Activity L. Similarly, TF_C = 7 units and TF_L
= 4 units. The float of 7 units could be used for Activity B without delaying completion of the total project;
however, Activity L would be delayed 4 days and no slack would be available for either Activity C or
Activity L.

Another advantage of time scale diagramming is that the status of individual activities can be seen. On
Figure 5-3 a status line for Time 15 is shown. The time where the status line crosses an activity indicates
the progress of that activity. For example, the status line crosses Activity L at Time 17, indicating that this
job is 2 units ahead of schedule, and crosses Activity G at Time 15 meaning that this operation is right on
time. Activity F is shown to be one day behind, and, since this job is on the critical path, the entire project
will be completed late unless the time can be made up elsewhere (on the remainder of Activity F or on
Activities K and/or M).

Time scale diagramming is not without its drawbacks. A significant one is that it is sometimes difficult to
select a suitable scale for drawing the diagram, particularly when durations of individual items vary
widely. Selecting a scale to show activity durations of 3 days and 135 days on the same chart is nearly
impossible. Thus, CPM's non-scaled diagrams can be an advantage at times.
Selection of the proper method will depend upon many factors including method of computation and the
background of the individuals to whom the work will be presented, as well as the variation in activity
duration. CPM and time scale diagramming will, however, yield the same results.

Economy Studies Using Arrow Diagramming Techniques
Once planning and scheduling has been completed, it may become apparent that economies can be
realized by altering the schedule or shifting resources. Reallocation of personnel and/or equipment and
the resulting schedule generally means increased activity costs since original schedules are usually
based on "normal" conditions, i.e., average levels of personnel and equipment. For example, reducing
time to complete the plumbing on a building may require more plumbers and/or overtime and, thus, more
money. Sometimes, however, the disadvantage of rescheduling can be outweighed by the advantages of
earlier completion.

Schedule Reduction
Consider the example project in Figure 5-4. Activity designations (A through L) and durations (days) are
given and ES/LS/EF/LF times are shown. Also presented are the increases in direct costs (such as
materials, equipment, and labor directly associated with the activity) required to complete each activity in
one day less time than originally planned. These increases in direct costs are often termed "crash" or
"crunch" costs. Assume that $1400/day in indirect costs (such as head office rental, principals' salaries,
and computer equipment not directly associated with any one project) can be saved for each day the
project finishes early. The task is to determine which activities can be economically reduced in duration.

To reduce overall project time we must reduce the duration of critical path activities; non-critical
operations can be ignored. Inspection of Figure 5-4 reveals that the critical activities are A, D, F, G, H,
and L. Activities A, G, and H can be excluded from consideration because they either cannot be reduced
or cost more to reduce 1 day than the $1400 that can be saved. Activity L can be reduced 1 day at a cost
of $1000 for an overall savings of $400 and a 13 day project completion time. Activity D can be reduced
1 day at a cost of $500 for a savings of $900. This second reduction has, however, created a second
critical path consisting of Activities B, I, K, and L. Therefore, further reductions in project time will
necessitate reductions in both critical paths. Combinations of activities to be simultaneously reduced
could be sought and identified, if possible, as long as the result is a net savings. Whether the savings is
for each individual day of reduction or for the cumulative effect of reduction depends on the situation at hand.

Example 5-x (under construction)
Two points should be made about the preceding discussion. One, increases in direct costs per day of reduction in activity duration are usually not linear. For example, if it costs $1000 to reduce an activity one day, it may cost $1250 to reduce a second day, $1750 to reduce a third day, and so on. Two, indirect costs are not the only possibility for saving money. Another is bonuses which Owners, particularly in the private sector, often pay for early completion.

Resource Leveling
Another type of economy study involving arrow diagramming techniques is resource leveling, the rescheduling of activities (generally non-critical ones) to allow for an evening out of personnel and equipment needs. The top part of Figure 5-5 is a time scale diagram for a project. Durations and ES/LS/EF/LF data are shown along with the number of workers required to complete each job in the left middle part of the figure. In the right middle of the figure is a chart which indicates when each activity may or must occur. Critical activities are cross-hatched indicating that no float is available; no special designation is shown for non-critical operations. The task is to schedule non-critical jobs so that the number of workers can be kept reasonably uniform.

The bottom of Figure 5-5 shows the results of the rescheduling as well as the resulting crew size. Comparing the middle and bottom parts of Figure 5-5, it can be seen that if all activities had gone at their earliest start times, a crew size of 9 persons would have been required on Day 2 and that only 2 workers would have been needed on Day 7. In the plan adopted, however, crew size ranged from a low of 4 to a high of 6. In most business operations, it is advisable to avoid large fluctuations in worker needs. There are usually financial incentives to avoiding short term hires and money can generally be conserved by stabilizing the work force.

Summary
Completing engineering projects "on time, within budget" requires considerable effort. Without proper planning and scheduling, doing so is highly improbable. Two major network diagramming techniques, the critical path method and time scale diagramming, have been described. Economy studies such as rescheduling to obtain shorter project periods and resource leveling for effective use of personnel and equipment have been introduced. These and other techniques can aid the project manager in finishing jobs in a timely and cost effective manner.

References
Project Management - Network Diagram

- **Critical Path Method - CPM**

  Bar chart weaknesses:
  - Does not show relationships between project activities
  - Does not identify activities which control a project's total duration (i.e., critical activities comprising critical path)
  - Does not relate delay or change of one activity to the entire project

  Network planning methods remedy bar chart weaknesses:
  - CPM - developed in late 1950s by industry to schedule maintenance & construction work utilizing computers
  - PERT - *Programme Evaluation & Review Technique* - developed in late 1950s by U.S. Navy to support the Polaris missile weapons system acquisition
  - CPM & PERT have much in common with PERT being somewhat more sophisticated with the use of probability concepts to deal with uncertain activity durations. CPM uses a single fixed duration for each activity.

  Types of CPM networks:
  - Activity - On - Arrow diagram or "arrow diagram"
  - Activity - On - Node diagram or "precedence diagram"
  - Arrow diagrams have been used most extensively and relate to most current software capability.

  Precedence diagrams are gaining in recognition and use

```
<table>
<thead>
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<th>Activity</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td>Event</td>
<td>Event</td>
<td>Event</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>
```

**Steps in critical path planning & scheduling.**
- Prepare list of project's activities.
- Estimate duration of each activity.
- Determine which activities immediately precede each activity.
- Determine which activities immediately follow each activity.
- Draw a network with the activities and events properly connected.
- Assign numbers to the events. Assure number at arrow's head is larger than number at arrow's tail.
- Prepare activities chart with activity name or letter, events, duration, ES, EF, LS, LF, TF & FF.
- Determine critical activities.
More CPM Terms and Relationships

D = Duration
ES = Earliest Start = Earliest finish time of preceding event
EF = Earliest Finish = ES + D
LS = Latest Start = LF - D
LF = Latest Finish = LS + D = Latest start time of following event
TF = Total Float = LF - EF = LS - ES
FF = Free Float = FF = ES (following activity) - EF (this activity)
TF = Time activities' start or finish can be delayed without delaying project completion.
FF = Time single activity's finish can be delayed without delaying the ES for a following activity.

Event Node

An event is the instant when an activity is started or completed. Events are represented by circles called nodes. They do not consume time. They occur when all the activities entering the node are completed.

Dummy Arrow

Dummy arrows are used to transfer logic from one event node to another in the net work. They are represented by broken arrows. A dummy arrow has zero duration and does not represent an activity. These symbols are used to represent the relations among the activities.

Activity A must be completed before Activity B can begin.

Activity C and D must both be completed before E can begin.

Activity F must be completed before Activities G and H can begin.
Activity N must be completed before Activity P can begin. Activities M and N must both be completed before Activity O can begin.

Activity-on-Arrow Diagramming

Arrow represents Activity "A".
Arrow is not to any scale.
Tail is activity start.
Head is activity finish.

Diagramming Example: Simple footing pierced by electrical service & water pipes

Activity

A - form
B - place rebar
C - place conduit/sleeves
D - pour concrete
E - strip forms
"A" precedes "B" or "C", Precedence
"B" & "C" may be accomplished concurrently, Concurrence
"D" must follow both "B" & "C", Succession

Dummy activity "C' " indicates "D" cannot be started until "C" is finished as well as "B" being finished.
So far, we have been in the planning phase of CPM.
Note, no times (durations) are present. Times come with the scheduling phase of CPM.
For computer support, programs require an activity's head event number to be greater than the tail event
number.

Therefore, change:

After preparing diagram, check the operations logic forward and backward to assure all precedence,
concurrence & succession relationships are valid.

Let's do some CPM scheduling.
Estimate activity durations; a crucial step to assure CPM reliability.
Bring team effort to bear for estimating durations.
Use average crew sizes, average productivity rates, etc., to develop average durations.
Post durations on the diagram (use 8-hour days).

Activity Duration
A - form 3
B - place rebar 4
C - place conduit/sleeves 1
C' - dummy arrow 0
D - pour concrete 6
E - strip forms 1

Critical Path is A-B-D-E, requiring 14 days duration. Etc.

Durations are referenced to the beginning or end of the work day. Pick one and be consistent. We will use
beginning of work day.
Information sought on which to base management decisions:
~earliest start time
~latest finish time
~float time
~critical path activities
This bar chart shows each activity’s start time, duration and finish time. The chart was constructed with each start being the earliest start time, so:
\[ \text{ES} + \text{D} = \text{EF} \]

What is the latest finish time for each activity that will still allow project completion by start of 13th day? Redraw the chart working backwards using:
\[ \text{LS} = \text{LF} - \text{D} \]

Note, Activities "F", "C" and "B" could all be delayed in finish (or start) by one day.

Note: "F" & "C" have one day free float, whereas, "F", "C" & "B" all have one day total float.

Total float is shared - e.g., if "B" used 1/2 day, "C" has 1/2 day left. Free float is a distinctive activity characteristic (not shaded).

**Critical Path Method - Notesheet**

*Points to remember:*
1. Scheduling tool
2. Project divided into tasks called activities
3. Arrows represent activities
4. Length, direction and shape of arrows is un-important
5. Name of activity above arrow
6. Duration of activity below arrow
7. Events represented by circles
8. Events are points in time - they have zero duration
9. Arrows may only connect events
10. Dummy (dashed) arrows have zero duration
Critical Path Analysis

Introduction
Businesses are constantly working on projects and from your own experience; you will know that management is very important. There are a number of methods that can be used to successfully manage a project, but we will be focusing on one particular aspect - time: extremely important to all businesses we acknowledge.

An unexpected delay or similar time crisis can put a business under a lot of pressure and perhaps it was all unforeseeable: evidence of poor project management?

So, how can you plan your time and identify where delays could occur, leaving you to make decisions to compensate for such situations? A useful tool to use would be the CRITICAL PATH ANALYSIS.

Critical Path Analysis - What is it?
The critical path analysis is a tool that illustrates the individual tasks of a project highlighting the expected starting and finishing tasks of each. More precisely, the critical path analysis can be used to:

• Estimate the minimum/maximum time that tasks will be started and completed
• Estimate the minimum time that the whole project will take to complete
• Identify if resources are not being used effectively
• Make aware any tasks that could create a possible delay

Ultimately, the critical path analysis will suggest which tasks are critical to keep on time anticipating that the delay in any one of the tasks will delay the whole project.

The critical path analysis is hard to explain in more detail without the use of diagrams, and so the working example in the following section will make it all more clear.

Example
XYZ Limited have decided to carry out some research to ultimately create a selling strategy for their new product. They have decided to create a questionnaire, which they will issue to the public personally. In addition, they will use a mail shot to send out a similar survey to get the opinions of those that live outside of the area.

Each task (A - F) has been given an expected completion time (in weeks). Time is crucial for XYZ Limited and so the Manager has requested a Critical Path Analysis of the project.

<table>
<thead>
<tr>
<th>Task</th>
<th>Description</th>
<th>Order/Logic</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Plan Primary Research</td>
<td>To be completed first</td>
<td>1 wks</td>
</tr>
<tr>
<td>B</td>
<td>Prepare Mail Shot (Postal Survey)</td>
<td>Start when A is complete</td>
<td>3 wks</td>
</tr>
<tr>
<td>C</td>
<td>Prepare Questionnaire</td>
<td>Start when A is complete</td>
<td>2 wks</td>
</tr>
<tr>
<td>D</td>
<td>Send and Wait for Mail Shot Replies</td>
<td>Start when B is complete</td>
<td>3 wks</td>
</tr>
<tr>
<td>E</td>
<td>Issue Questionnaire</td>
<td>Start when C is complete</td>
<td>3 wks</td>
</tr>
<tr>
<td>F</td>
<td>Compile and Analyze Results</td>
<td>Start when D &amp; E is complete</td>
<td>2 wks</td>
</tr>
<tr>
<td>G</td>
<td>Plan Selling Campaign</td>
<td>Start when D, E &amp; F is complete</td>
<td>2 wks</td>
</tr>
</tbody>
</table>
Now, take a good look at the diagram below. Each circle (Node) will be used to enter specific data. The numbers currently in the nodes (1 - 7) are only there to make following the diagram easier - nothing more. The arrows represent the tasks and each is given their respective completion times.

Before we move on, look at the above table again and ensure that you understand what is being said by linking it back to the diagram. Basically, all tasks cannot start until the previous task has been completed. This is not true for tasks B and C which can start at the same time - this is the only tricky area.

First of all, we have to identify the **earliest starting time (est)** for each task. This figure is then entered into the top right hand segment of the node. To work out the est's, we move from left to right on the diagram. The diagram below shows all the est's for the project and by hovering the mouse over a node, it will show how this figure was obtained.

The next stage is explained on the following page.

### Critical Path Analysis Part 2

Secondly, we have to work out the **latest finish time (lft)** for each task. This is worked out similar to the est's except we subtract figures and work from right to left on the diagram. The lft is then placed inside the bottom right hand segment of the node.

Ignore the red line for now, this will be explained later. Remember - from right to left.

Now the est's and the lft's have been calculated, we can determine the **critical path**. This is found by recognizing those nodes where the est = lft.
By looking at the above diagram, all the nodes have equal est and lft except for number 4. By acknowledging this, we can identify the critical path as:

A - B - D - F - G (highlighted by the red line)

In other words, there must be no delays in completing these tasks, otherwise the project completion time will be also be delayed beyond the expected 11 weeks. This is not true for the tasks that do not lie on the critical path, as explained below.

So, what can we determine from those tasks that do not lie on the critical path - C and E? Well, either task C or E can be delayed by 1 week without affecting the completion time of the project. This is called the float.

There are two types of float - the total float and the free float. Only those tasks that are not on the critical path will have a float, as you will see.

**Total Float**

The total float is the accumulated float up to the specific task. This is worked out by subtracting the est and the duration from the lft:

$$\text{Total Float} = \text{LFT} - \text{EST} - \text{Duration}$$

<table>
<thead>
<tr>
<th>Task</th>
<th>Duration</th>
<th>Est</th>
<th>Lft</th>
<th>Total Float (wks)</th>
<th>Free Float (wks)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0 : (1 - 0 - 1)</td>
<td>0 : (1 - 0 - 1)</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>0 : (4 - 1 - 3)</td>
<td>0 : (4 - 1 - 3)</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>1 : (4 - 1 - 2)</td>
<td>0 : (3 - 1 - 2)</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>4</td>
<td>7</td>
<td>0 : (7 - 4 - 3)</td>
<td>0 : (7 - 4 - 3)</td>
</tr>
<tr>
<td>E</td>
<td>3</td>
<td>3</td>
<td>7</td>
<td>1 : (7 - 3 - 3)</td>
<td>1 : (7 - 3 - 3)</td>
</tr>
<tr>
<td>F</td>
<td>2</td>
<td>7</td>
<td>9</td>
<td>0 : (9 - 7 - 2)</td>
<td>0 : (9 - 7 - 2)</td>
</tr>
<tr>
<td>G</td>
<td>2</td>
<td>9</td>
<td>11</td>
<td>0 : (11 - 9 - 2)</td>
<td>0 : (11 - 9 - 2)</td>
</tr>
</tbody>
</table>

E.g. For task C, the total float = 4 - 1 - 2 = 1 (week)

**Free Float**

The free float is worked out by subtracting the est at the start of the task and the duration, from the est at the end of the task:

$$\text{Free Float} = \text{End EST} - \text{Start EST} - \text{Duration}$$

i.e. Free Float = End EST - Start EST - Duration

So, again for task C, the free float = 3 - 1 - 2 = 0
This is showing that task C can be delayed (like all tasks), but it will have an effect on the start time of the next task. All tasks that calculate zero has the same rule applying.

But, what about those tasks that calculate a figure other than zero? If we work out the free float for task E, we get:

Free Float = 7 - 3 - 3 = 1 week

Now, this means that task E can be delayed by 1 week without having an effect on the start time of the next task (F). Any delays over this time would only then affect the proceeding task. For example, if task E was delayed by 2 weeks, it would delay the start time of task F by 1 week - 1 week compensated by the float, and the other causing 1 week delay.

Finally, let's now look at the floats for all the tasks.

By looking at the table, those tasks without a 'total float' (i.e. zero) are considered 'critical' and coincidentally are found on the critical path. It is therefore important that these tasks are not delayed in order to complete the project on time as planned (11 weeks).

Acknowledging and integrating float is very important. For example, those tasks that do carry float may have resources (labour, capital, equipment, etc) that could be used elsewhere to complete other tasks quicker.

Also, for those tasks that do carry float, any delays can be accepted unless it is sufficient enough to eliminate the float completely. In such case, at sometime previous to the current task, a major problem or issue has occurred - investigate and act immediately.

I'm Lost!
We do accept that this area can be confusing in places, and so if you are unsure that you can now go and complete a similar analysis for your project, visit the following link: A new window will open and you will be directed away from this web-site. Here, you can enter your details for the project on-line and let the computer calculate the appropriate figures.

Further Reading
This article went through the basics of the critical path analysis (believe it or not!) and so, if you believe that such a tool can be useful in your business, why not read further.
Critical Path Method (CPM) (using Microsoft Excel)

Introduction to Network Analysis of Projects and CPM

The Critical Path Method (CPM) is one of several related techniques for doing project planning. CPM is for projects that are made up of a number of individual "activities." If some of the activities require other activities to finish before they can start, then the project becomes a complex web of activities.

CPM can help you figure out:
- how long your complex project will take to complete
- which activities are "critical," meaning that they have to be done on time or else the whole project will take longer

If you put in information about the cost of each activity, and how much it costs to speed up each activity, CPM can help you figure out:
- whether you should try to speed up the project, and, if so,
- what is the least costly way to speed up the project.

Activities

An activity is a specific task. It gets something done. An activity can have these properties:
- names of any other activities that have to be completed before this one can start
- a projected normal time duration

If you want to do a speedup cost analysis, you also have to know these things about each activity:
- a cost to complete
- a shorter time to complete on a crash basis
- the higher cost of completing it on a crash basis

CPM analysis starts after you have figured out all the individual activities in your project.

CPM Analysis Steps, By Example

This document describes the steps for doing CPM analysis for this course. The steps will be illustrated by two examples. I recommend that you work through the examples, so that you can follow the steps yourself when you do the homework.

Example 2 is especially valuable for you to work through. Excel has bugs that vary from version to version. By working through Example 2, and comparing what you get with what I got, you can find out which bugs apply to you and how to work around them when you do the assignment.

- Example 1: Activities, precedence, and times
  This first example involves activities, their precedence (which activities come before other activities), and the times the activities take. The objective is to identify the critical path and figure out how much time the whole project will take.
**Example 1 Step 1: List the activities**

CPM analysis starts when you have a table showing each activity in your project. For each activity, you need to know which other activities must be done before it starts, and how long the activity takes.

Here's the example:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Description</th>
<th>Required Predecessor</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Product design</td>
<td>(None)</td>
<td>5 months</td>
</tr>
<tr>
<td>B</td>
<td>Market research</td>
<td>(None)</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>Production analysis</td>
<td>A</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>Product model</td>
<td>A</td>
<td>3</td>
</tr>
<tr>
<td>E</td>
<td>Sales brochure</td>
<td>A</td>
<td>2</td>
</tr>
<tr>
<td>F</td>
<td>Cost analysis</td>
<td>C</td>
<td>3</td>
</tr>
<tr>
<td>G</td>
<td>Product testing</td>
<td>D</td>
<td>4</td>
</tr>
<tr>
<td>H</td>
<td>Sales training</td>
<td>B, E</td>
<td>2</td>
</tr>
<tr>
<td>I</td>
<td>Pricing</td>
<td>H</td>
<td>1</td>
</tr>
<tr>
<td>J</td>
<td>Project report</td>
<td>F, G, I</td>
<td>1</td>
</tr>
</tbody>
</table>

**Example 1 Step 2: Draw the diagram**

Draw by hand a network diagram of the project that shows which activities follow which other ones. This can be tricky. The analysis method we'll be using requires an "activity-on-arc" (AOA) diagram. An AOA diagram has numbered "nodes" that represent stages of project completion. You make up the nodes' numbers as you construct the diagram. You connect the nodes with arrows or "arcs" that represent the activities that are listed in the above table.

Some conventions about how to draw these diagrams:
- All activities with no predecessor come off of node 1.
- All activities with no successor point to the last node, which has to have highest node number.
In this example, A and B are the two activities that have no predecessor. They are represented as arrows leading away from node 1.

J is the one activity that has no successor, in this example. It therefore points to the last node, which is node 8. If there were more than one activity with successor, all of those activities’ arrows point to the highest number node.

Students sometimes make the mistake of creating a diagram with several starting or ending nodes. Don’t do this.

The trickiest part for me of building the above diagram was figuring what to do with activity H. I had drawn an arrow for activity B coming off node 1 and going to mode 3. I had later drawn an arrow for activity E coming off node 2 and going to node 6. Since H requires both B and E, I had to erase my first E arrow and redraw it so it pointed to the same node 3 that B did. H then comes off of node 3 and goes to node 6.

When designing these diagrams, work in pencil.

**Example 1 Step 3: Set up the CPM spreadsheet**

There are specialized commercial programs for doing CPM analysis. Rather than purchase and learn one of those, we'll leverage the spreadsheet knowledge we already have. We will use one freeware program written for this course and made available to you through the Internet.

Start up a new blank spreadsheet. If you are viewing this document on the web, minimize your browser window and then start Excel. That way you can switch from one to the other by pressing Alt+Tab.

In a blank spreadsheet, type the word "Activities" in cell A1. In row 2, type the names of the activities, or their letters. (To make my spreadsheet screen shots fit better on these pages, I set the column widths to 4. You do not have to do this.)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>F</td>
<td>G</td>
<td>H</td>
<td>I</td>
<td>J</td>
</tr>
</tbody>
</table>

In row 3, type "Nodes". In row 4, type in each activity's start node -- where the tail of its arrow is. Below that, in row 5, type each activity's end node -- where the head of its arrow is. Do this carefully. Mistakes here mess up everything that follows.

To the right, in K2 and K3, type the words "Start" and "End" to label those rows.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>F</td>
<td>G</td>
<td>H</td>
<td>I</td>
<td>J</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>6</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>7</td>
<td>7</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>
In cell A6, type "Times". In row 7, type the time each activity takes. Then, select the range of cells containing the node numbers and copy it to the clipboard.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Activities</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>F</td>
<td>G</td>
<td>H</td>
<td>I</td>
<td>J</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Nodes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>6</td>
<td>7</td>
<td>Start</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>7</td>
<td>7</td>
<td>6</td>
<td>7</td>
<td>6</td>
<td>End</td>
</tr>
<tr>
<td>6</td>
<td>Times</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**Example 1 Step 4: Use Pathfind to get the paths**

Pathfind is a computer program that helps you find and enter into the spreadsheet all of the possible paths through your diagram along the arrows from the first node to the last. You could do this by hand, of course. This diagram shows the four possible paths in this example.

The four paths are A D G J, A C F J, A E H I J, and B H I J. We'll code them in the spreadsheet with a matrix of 0's and 1's. Rather than do this all by hand, we'll get Pathfind help do it.

To use Pathfind, start up your Internet connection and your browser (unless, of course, you are reading this document on the Internet already).

Go to http://hspm.sph.sc.edu/Courses/J716/CPM/Pathfind.html

Loading this html file into your browser starts Pathfind, which is a Java applet that runs inside your browser. When Pathfind is loaded:

1. Click in Pathfind's upper text area.
2. Paste the range you just copied from your spreadsheet into that upper text area. (Click in the text area and press Ctrl+V or Shift+Insert.)
3. Click on Pathfind's button. Pathfind's lower text area will give you a block of numbers, all highlighted so you can copy them.
4. Copy the highlighted numbers to the clipboard for pasting later into your spreadsheet. (Ctrl+C or Ctrl+Insert copies what is highlighted.)

You can now close the Pathfind web page, if you wish.

**Example 1 Step 5: Paste the path information into your spreadsheet**
When you're done with Pathfind, go back to your spreadsheet. Move your cell selector to cell A8. Type "Paths" in that cell. Then move the cell selector to A9, as shown here:

```
<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Activities</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>F</td>
<td>G</td>
<td>H</td>
<td>I</td>
<td>J</td>
</tr>
<tr>
<td>3</td>
<td>Nodes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
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<td>4</td>
<td>5</td>
<td>3</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>7</td>
<td>7</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>Times</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>Paths</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

Paste to that cell, to see this:

```
<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
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</tbody>
</table>
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The pasted cells are all 0's and 1's. Each row represents a path. The 1's indicate which activities are in that particular path. For example, row 9 (cells A9:J9) has 1's under activities A, C, F, and J. This says that this path includes activities A, C, F, and J. This corresponds to the path through the middle of the diagram that goes: 1 -A-> 2 -C-> 4 -F-> 7 -J-> 8.

The diagram above shows four paths from node 1 to node 8. Sure enough, Pathfind gives you four rows of 0's and 1's, one row for each path.

**Example 1 Step 6: Calculate the paths' times**
Move the cell selector to K9. Type =SUMPRODUCT(A9:J9,$A$7:$J$7) in that cell. For Quattro Pro and Lotus, type @ instead of =.

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Example 1 Step 7: Identify the critical path

The critical path is the path that takes the longest. In this example, the critical path is the one in row 10, which takes 13 weeks. The project will therefore take 13 weeks, if everything is done on schedule with no delays. The time a project takes is equal to the time of its critical path.
The 1's in row 10 tell us that the critical path is 1 -A-> 2 -D-> 4 -G-> 7 -J-> 8. As managers, we must be sure that activities A, D, G, and J are done on time. If any of those activities is late, the project will be late.

Other paths are not critical because they can waste some time without slowing the project. For example, activity C, in row 9's path, can take up to two extra weeks and not hold up the project. To make it easier to see what activities are in each path, go to cell A14. Type =if(A9=1,A$2,"") there.

The letter A should appear in cell A14.

This =if(A9=1,A$2,"") function works this way: Inside the parentheses are three expressions separated by commas. The first expression (A9=1) is something that can be either true or false. If the expression is true, the second expression (A$2) is shown in the cell. Otherwise, the third expression ("") is shown in the cell.

In A14, the expression A9=1 is true, so the cell shows what is in A2, which is the letter "A". If A9 had not contained a 1, the A14 would have shown a blank, which is what "" means.

Copy A14 to the clipboard. Then, starting in A14, select a range of cells that goes over to column J and down four rows. The selected range should be the same size as the space that the paths' 1's and 0's take up.

Paste. You should get this:

<table>
<thead>
<tr>
<th>A</th>
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</tbody>
</table>

Now you can see which activities are in each path. If your results do not look like the above, make sure that there is one $ in your formula, and that it's in front of the 2 and not in front of the A.

Go to cell J13 and type "Max". Then go to cell K13. Type =MAX(K9:K12) to display the longest path time.
Move to cell K14 and type =IF(K9=K$13,"Critical","") there.

Move to cell K14 and type =IF(K9=K$13,"Critical","") there.

This will put the word "Critical" next to a path whose time equals the maximum of all the path times. Otherwise, it will put in a blank, as it does here, because the 11 in K9 does not equal the 13 in K13.

Copy K14 to the clipboard. (It will seem strange to copy what appears to be an empty cell, but do it anyway.) Select cells K14 to K17, and paste.
You're done! You've found the time the project will take, and you have identified the critical path, which tells you which activities must be done on time to make the project finish in the least time.

Take a moment to admire your work before plunging in to Example 2.

- **Example 2: Costs and Crash Costs**
  This second example incorporates costs and the possibility of spending money to speed up the project. Our objective is to determine how quickly we should complete each activity, and thus the how long the project as a whole should take. The presumption is that there is some reward for getting finished sooner. We must decide whether the reward is worth earning, and, if so, what is the best way to earn it.

This example also shows how to use a dummy activity. A dummy activity is an activity that you add to the original activities list. A dummy activity takes no time, and it has no cost. You'll learn why you sometimes need such a thing in a CPM model.

**Example 2 Step 1: List the activities**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Required Predecessor</th>
<th>Normal Time</th>
<th>Normal Cost</th>
<th>Crash Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(None)</td>
<td>3 weeks</td>
<td>$3000</td>
<td>2 weeks</td>
</tr>
<tr>
<td>B</td>
<td>(None)</td>
<td>4</td>
<td>$4000</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>(None)</td>
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</tbody>
</table>
Example 2 Step 2: Draw the diagram
To start the network diagram, we notice that A, B, and C are the three activities with no predecessor. They all come off of node 1. A can go node 1 to node 2. B can go from node 1 to node 3. C also starts at node 1. D requires A, so D starts at node 2. Here's what we have so far:

Now for activity E. Activity E requires a special trick. The problem is where E should start. E requires A and B. A ends at node 2 and B ends at node 3. E is not allowed to start from both nodes 2 and 3. Activities can have only one start node and only one end node.

What do we do about E? You might consider connecting both A and B to node 2, but that would mess up Activity D. If both A and B were to run from node 1 to node 2 and D came off of node 2, that would be saying that D requires B as well as A. D is supposed to require A, not also B. Here is the solution:

The solution is to add a dummy activity that runs from node 2 to node 3, as shown in the diagram. Then start E at node 3.

If E starts at node 3, it means that E requires B and the dummy activity, the two activities that come in to node 3. The dummy activity, because it starts at node 2, requires A. This makes E require B and A. That is what we want! Meantime, D starts at node 2, so D only requires A. All the requirements are satisfied!
Dummies activities add nothing to the time or the cost. Their purpose is to allow you to represent complex relationships among activities.

**Example 2 Step 3: Set up the CPM spreadsheet**
This time we have 7 activities. That's the six lettered activities plus the one dummy activity, which we'll call Dummy.

These instructions will create the spreadsheet from scratch. Adapting the spreadsheet from example 1 is possible, but tricky, because example 2 has fewer columns and more rows than example 1.

If you start with a blank spreadsheet, move the cell selector to A1 and type "Activities". In row 2, type the activities' letters or names. Put the Dummy at the right end of row 2, for two reasons:
1. The other activities' names match will their column letters.
2. It will be easier to fill in the Solver Parameters box when we get to that stage.

In row 3, type "Nodes".
In row 4, type in the activities' start nodes.
In row 5, type in the activities' end nodes.

<table>
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Type "Times" in cell A6.

In row 7, type each activity's normal time. The dummy's time is 0.
In row 8, type each activity's crash time. The dummy's crash time is 0.
Copy row 7 and paste it into row 9. Row 9 will be the variable cells when we do the optimization later. Be sure to copy the numbers themselves from row 7 to row 9. Don't put a formula like =A7 in A9.

Label each of the Times rows by typing "Normal" in H7, "Crash" in H8, and "Actual" in H9. *(Note: When you do the homework, your number of columns will be different. These labels go in the column to the right of the last activity's information. Whatever column that is, use its letter in place of "H" in all the following instructions and formulas.)*

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When we do the optimization, we'll set a maximum time for the project and tell the spreadsheet to find the combination of numbers in row 9 that completes the project within that time for the lowest possible cost.

To do that, we need the costs. In cell A10, type "Costs".
In row 11, put each activity's normal cost. In H11, type "Normal".
In row 12, put each activity's crash cost. In H12, type "Crash".
In H13, type "Actual".
Type "0" in G13 for the Dummy's actual cost.
(When you do the homework, your column letters will differ.)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
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<td>13</td>
<td>=A11+(A7-A9)/(A7-A8)*(A12-A11)</td>
<td>0 Actual</td>
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</table>

The diagram above also shows a formula we'll put in A13.
Row 13 will have formulas to calculate the actual cost for each activity. Each activity's actual cost depends on how much it is sped up, or "crashed." We assume a linear relationship between speed-up and cost. So, for example, if Activity A can be shortened by 2 weeks at an added cost of $2000, we assume that it can be shortened by 1 week for an added cost of $1000.

The formula to implement this goes first A13. Here it is:
=A11+(A7-A9)/(A7-A8)*(A12-A11)

If you are viewing this in a web browser, you can select and copy the above formula right off of the screen. Then paste it into cell A13 of your spreadsheet.

The logic of the formula:
(A7-A9) is the difference between the normal time and the actual time we use for activity A. This difference is how much time we are saving by speeding up activity A.

(A7-A8) is the difference between the normal time and the crash time. This is the most time we could save by speeding up activity A.

(A7-A9)/(A7-A8) is how much time we are actually saving, as a fraction of how much time we could save, for activity A. In other words, it is the proportion of the possible time savings that we are actually using.
(A12-A11) is the difference between the crash cost and the normal cost for activity A. This difference is how much cost would go up if we shortened activity A's time as much as possible. Multiplying these, to get \( \frac{(A7-A9)}{(A7-A8)}(A12-A11) \), tells us additional cost we are incurring by shortening activity A's time from its normal time to the actual time we chose. This embodies the linearity assumption -- that if we go part way between the normal time and the crash time, our cost will be that same part way between the normal cost and the crash cost. The full formula, \( A11+(\frac{(A7-A9)}{(A7-A8)})(A12-A11) \), adds that additional cost to A11, the cost of doing the activity in normal time. This gives us the cost of doing activity A in the amount of time in A9.

Don't paste the formula to cell G13, because that would give you a division-by-zero error. Similarly, if any of your other activities cannot be sped up (the crash time equals the normal time), put the normal cost number in the cell in row 13, not the formula.

Once that formula is in, copy cell A13 to the clipboard. Select cells A13:F13, and paste.

<table>
<thead>
<tr>
<th>A</th>
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Right now, it may look like the formula isn't doing much, because the Actual costs match the Normal costs. This is because the Actual times match the Normal times. Later, when we shorten the project, this will change.

Now, select the range of node numbers, being sure to include the dummy activity's node numbers.

<table>
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<tr>
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Copy this range to the clipboard.
Example 2 Step 4: Use Pathfind to get the paths
Go to http://hspm.sph.sc.edu/Courses/J716/CPM/Pathfind.html and paste in the node numbers you just copied. Follow Pathfind’s instructions for copying its reply.

Example 2 Step 5: Paste the path information into your spreadsheet
Go back to your spreadsheet. In cell A14, type "Paths". Then move your cell selector to A15, as shown here:

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Paste to that cell, to see this:

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As with the first example, each row represents a path. The 1’s indicate which activities are in that particular path. For example, row 15 has 1’s under activities A and D. This represents the
path 1 -A-> 2 -D-> 5 at the top of the diagram. There are four possible paths from node 1 to node 5, so you have four rows of 0's and 1's.

**Example 2 Step 6: Calculate the paths' times**

In cell H15, put =SUMPRODUCT(A15:G15,$A$9:$G$9). (When you do the homework, your ending column letter may differ. You want this formula to cover all of the activities.)

Copy that cell and paste it to H15:H18. (When you do the homework, you will paste to a different range of cells. You will have a different number of activities and a different number of paths.)

We can now see how long each path takes.
Example 2 Step 7: Identify the critical path
The critical path is in row 15, 1 -A-> 2 -D-> 5. It’s the path with the longest time.

To make it easier to see which activities are in each path, go to cell A20 and type =if(A15=1,A$2,""") (Notice that the $ sign is before the 2, not before the A.) This should put an "A" in A20.

Copy cell A20 to the clipboard. Select the range A20:G23. Paste.

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</table>

You now should have four rows showing the letters of the activities that are in each path. Go back up to cell G19 and type "Max". Go to cell H19 and type =max(H15:H18) This shows how long the slowest path takes.

Go down to cell H20. Put in =IF(H15=H$19,"Critical","""). In this example, "Critical" appears in cell H15, because this first path is the critical path. (In your homework, if the first path does not happen to be critical, this cell will appear blank.)

Sure enough, the first path with activities A and D, is the only one labelled "Critical".

Example 2 Step 8: Total cost formula
Go up to cell E14 and type "Total actual cost:“. (In your spreadsheet, you may be able to use F14. The idea is to leave enough room so that this label does not spill over into the H column.) In cell H14, add up the actual costs of all of the activities. The formula is =SUM(A13:G13)
When you complete the formula, you’ll see that if we use all normal times for all activities, the total cost is $24,000.

**Example 2 Step 9: Fill in the optimizer form**
Crash analysis is linear programming in disguise. To perform crash analysis, we use the Solver tool. From *Excel*'s menu, select Tools, then Solver.

Fill in the Solver Parameters box as shown here. *(For the homework, modify the formulas so they cover the rows and columns that you have.)*

![Solver Parameters](image)

- The Target Cell is H14, the total cost.
- Click on Min. Very important and easy to overlook. We want to find the least-cost way to speed up the project.
- The Changing Cells are the Actual times, in A9:G9.
- The constraints, which you add by clicking the Add button, are:
  - A9:G9 <= A7:G7 All the Actual times must be less than or equal to the Normal times. We assume that we don’t save any money by going slower than the Normal time. *(Notice again that these formulas exclude the Dummy.)*
  - A9:G9 >= A8:G8 All the Actual times must be greater than or equal to the crash times. The crash times are, by definition, the fastest possible times for each activity.
  - H15:H18 <= 10 The slowest path can take no more than 10 weeks. 10 is chosen because it’s one less than 11, the normal completion time. Later, you can change this to 9, 8, etc., to see what happens when you try to finish the project in shorter and shorter times.

If you are using Excel 2000 or earlier, click on Options. Click the checkbox for assuming a linear model.
Excel 2007 and 2003 do not need this, but Excel 2000 seems to. If you do not check this box, Excel 2000 may tell you at some point that there is not a feasible solution when actually there is one. Some older Excel versions, though, give wrong answers if you check this box. For older versions of Excel, my advice is to first try telling it that this is a linear model, as shown here, and see what happens when you Solve. If it won't give you a solution, uncheck the linear model checkbox and solve again.

**Example 2 Step 10: Solve**
Click on OK in the Solver Options dialog box. Click on Solve in the Solver Parameters dialog box.

If an error message appears, bring back up the Solver Parameters dialog box.
- Make sure that Min is checked. A mistake here can cause an "unbounded solution" error.
- Verify that the target cell and the changing cells are correct.
- Verify that all the constraints are correct, with greater-thans and less-thans going in the right directions. Mistakes here can cause "unbounded solution," "non-linear," and "no feasible solution" errors.
- If tried all of those, without avail, you can try the following, which are workarounds for bugs in some versions of Excel.
  - If you get a message that there is no feasible solution, try changing the Assume Linear Model option. That is, click Solver's Options button, then check Assume Linear Model if it's unchecked, or uncheck it if it's checked.
  - If you still get the no-feasible-solution message, change the Solver Parameters as shown here to exclude the dummy activity from the Changing Cells and the Constraints.
The ranges for the Changing Cells and the Constraints stop at column F. Column G has the dummy activity.

Excel 2000 and later should not require this modification. Please let me know if you have to resort to this.

If everything is correct, a dialog box will ask you what reports you want. You can request whatever reports if you like, but I won't do anything with them here.

The result should be:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Activities</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>F</td>
<td>Dummy</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Nodes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>2 Start</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>3 End</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Times</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>8</td>
<td>3</td>
<td>5</td>
<td>0 Normal</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>2</td>
<td>3</td>
<td>0 Crash</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>3</td>
<td>5</td>
<td>0 Actual</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Costs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>3000 4000 5000 5000 3000 4000</td>
<td></td>
<td>0 Normal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>5000 6000 6000 6000 4000 6000</td>
<td></td>
<td>0 Crash</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>3000 4000 5000 5500 3000 4000</td>
<td></td>
<td>0 Actual</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Paths</td>
<td></td>
<td>Total actual cost</td>
<td>24500</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>17</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>19</td>
<td>Max</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>20</td>
<td>A</td>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Critical</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>B</td>
<td>E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>C</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Critical</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>A</td>
<td>E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Dummy</td>
<td></td>
</tr>
</tbody>
</table>
To get finished in 10 weeks (H19 now has 10), we'll have to spend $24,500 (H14 now has 24500).

As manager, you'll be busier. You now have two critical paths to worry about, 1 -A-> 2 -D-> 5, and 1 -C-> 4 -F-> 5. If any of the four activities in those paths is late, the project will take more than 10 weeks.

Let's add three more rows, to make it easier to see which activities have speeded up and at what extra cost:

Go to cell A24 and type "Crashed by how much"
Then go to A25 and put =A7-A9 there. This is the Actual time minus the Normal time.
Go to cell A26. Type in =A13-A11. This is the difference between the current Actual cost and the Normal cost.


We see that Activity D has been shortened by 1 week, at an extra cost of $500.

**Example 2 Step 11: Economic Analysis**

Let's make up an economic problem to solve. Imagine that the Example 2 project is being done on a contract, with a scheduled completion time of 8 weeks. There is a $2500 per week penalty for being late. There is also a $1000 per week bonus for being early. Our objective is to find the
best (least cost) schedule for the project. Does it pay to be on time, or are we better off paying some penalty? It is worth it to go for the bonus?

Here are our results so far. An explanation follows:

<table>
<thead>
<tr>
<th>Weeks</th>
<th>Project cost</th>
<th>Penalty cost</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>$24,000</td>
<td>$7,500</td>
<td>$31,500</td>
</tr>
<tr>
<td>10</td>
<td>24,500</td>
<td>5,000</td>
<td>29,500</td>
</tr>
</tbody>
</table>

If we use normal times, the project takes 11 weeks, which runs over the schedule by 3 weeks. We lose $7500 in penalties. The total project cost is $24,000 + $7,500 = $31,500.

If we crash by 1 week, the project takes 10 weeks. Our penalty cost is $5,000. Direct project cost is $24,500, as we just saw. The total is $29,500. This is less than $31,500, so ten weeks is better than eleven weeks.

To try getting done in 9 weeks, go back to the Solver (step 9). Change the H15:H18 <= 10 constraint to H15:H18 <= 9. Solve. The cost rises to $26,500. The penalty for being one week late is $2,500. Total cost is $26,500 + $2,500 = $29,000. This is less than $29,500, so nine weeks is better than ten weeks.

<table>
<thead>
<tr>
<th>Weeks</th>
<th>Project cost</th>
<th>Penalty cost</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>$24,000</td>
<td>$7,500</td>
<td>$31,500</td>
</tr>
<tr>
<td>10</td>
<td>24,500</td>
<td>5,000</td>
<td>29,500</td>
</tr>
<tr>
<td>9</td>
<td>26,500</td>
<td>2,500</td>
<td>29,000</td>
</tr>
</tbody>
</table>

Notice that cutting the second week added more to cost (second column) than cutting the first week. Cutting one week increased project cost by $500. Cutting the second week increased project cost by $2000, from $24,500 to $26,500. The law of diminishing returns is at work here.

Should we save three weeks and be on time? To try getting done in 8 weeks, go back to the Solver. Change the H15:H18 <= 9 constraint to H15:H18 <= 8. Solve. You should find that the cost is $30,000. There is no penalty, but $30,000 is more than $29,500, so we lose money by being on time. We are better off being one week late and paying the penalty.

What about saving four weeks and being early? Thanks to the law of diminishing returns, we don't need to consider shorter project durations than 8 weeks. Going from 9 weeks to 8 added $3,500 to cost. The law of diminishing returns implies that going from 8 weeks to 7 will add at least $3,500 more to cost. That is more than the $1000 bonus, so we know 7 weeks is a loser.

If the bonus is linear (in other words, if we get the same bonus for each additional week that we are early) then the law of diminishing returns implies that we can stop our analysis as soon as the total cost, including the bonus, starts to rise.

Here is the whole table. The bonus for being early is treated as a negative penalty.
We conclude that our optimal production schedule is 9 weeks. It has the least total cost.

**CPM Steps Summary**
CPM helps you identify a complex project's critical paths. You can find how long a project will take and which activities must be on time. If you also have information about costs and crash costs and times, CPM helps you determine how long the project should take, and which activities should be sped up ("crashed"). As we are doing it in this class, the steps are:
1. Have a list of the activities.
2. Draw the network diagram.
3. Put activity names, node numbers, times, and costs in a spreadsheet.
4. Use Pathfind to generate code for the paths.
5. Put the path information into the spreadsheet.
6. Calculate the paths' times.
7. Identify the critical paths, and the activities in each path.
8. Set up the formula to calculate the project's total cost.
9. Fill in the Tools | Solver... form.
10. Solve, and fix errors, if any.
11. For an economic analysis, change the maximum time constraint and solve again. Repeat until costs, including penalties and bonuses, start to go up.

**Exercise:**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Required Predecessor</th>
<th>Normal Time</th>
<th>Normal Cost</th>
<th>Crash Time</th>
<th>Crash Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(None)</td>
<td>8</td>
<td>$4000</td>
<td>6</td>
<td>$6000</td>
</tr>
<tr>
<td>B</td>
<td>(None)</td>
<td>5</td>
<td>1500</td>
<td>4</td>
<td>2000</td>
</tr>
<tr>
<td>C</td>
<td>(None)</td>
<td>6</td>
<td>2500</td>
<td>4</td>
<td>3000</td>
</tr>
<tr>
<td>D</td>
<td>A</td>
<td>4</td>
<td>1800</td>
<td>3</td>
<td>2000</td>
</tr>
<tr>
<td>E</td>
<td>A, B</td>
<td>6</td>
<td>1000</td>
<td>5</td>
<td>1200</td>
</tr>
<tr>
<td>F</td>
<td>C</td>
<td>7</td>
<td>2000</td>
<td>5</td>
<td>3000</td>
</tr>
<tr>
<td>G</td>
<td>A</td>
<td>5</td>
<td>3000</td>
<td>3</td>
<td>6000</td>
</tr>
<tr>
<td>H</td>
<td>D, E, F</td>
<td>8</td>
<td>4500</td>
<td>5</td>
<td>9000</td>
</tr>
<tr>
<td>I</td>
<td>C</td>
<td>9</td>
<td>6000</td>
<td>4</td>
<td>10000</td>
</tr>
<tr>
<td>J</td>
<td>D, E, F</td>
<td>6</td>
<td>6000</td>
<td>4</td>
<td>8000</td>
</tr>
<tr>
<td>K</td>
<td>G, H</td>
<td>4</td>
<td>2000</td>
<td>3</td>
<td>2600</td>
</tr>
<tr>
<td>L</td>
<td>D, E, F</td>
<td>6</td>
<td>3000</td>
<td>3</td>
<td>9000</td>
</tr>
<tr>
<td>M</td>
<td>I, J</td>
<td>4</td>
<td>8000</td>
<td>2</td>
<td>12000</td>
</tr>
</tbody>
</table>
The scheduled completion time is 25 weeks. You must pay a $1500 penalty for every week you are late. You get a $1000 bonus for every week you are early.

1. Show your network diagram.
2. Find the normal completion time and the critical path.
3. Determine the schedule that minimizes your total cost for this project, including any penalty or bonus.
   a. How did you decide when to stop trying shorter and shorter completion times?
   b. How many weeks total should the project take?
   c. What will your total cost be?
   d. Which activities will be shortened from their normal times, and by how much?
   e. Which activities are critical to the least cost schedule?

When you're ready, check your answers with the Answer Checker for assignment 12.
Critical Path Network (Tutorial)

This tutorial is designed for engineering students who plan to use the Critical Path Method (CPM) to schedule and monitor their team's progress toward completion of a project.

OBJECTIVES:
After completing the critical path tutorial, you should be able to
- Define terminology related to the CPM
- Construct a network diagram for a given set of activities and corresponding times
- Recognize network diagrams that violate one or more of the CPM rules
- Calculate TE, TL, and S for a given network
- Apply CPM to identify a network's critical path(s).

Network Drawing
A network consists of interrelated and interdependent activities and events.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Duration</th>
<th>Path to D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-B</td>
<td>2</td>
<td>A-B-D</td>
</tr>
<tr>
<td>Events A, B, and C</td>
<td></td>
<td>A-C-D</td>
</tr>
<tr>
<td>The arrows show direction and connection.</td>
<td></td>
<td>A-B-C-D</td>
</tr>
</tbody>
</table>

- You can learn a lot by studying the network diagram. For example:
- You can go from A to B; you can't go from A directly to E.
- The shortest path to D takes 6 time units.
- You can't go from B to A.
- The longest path to D takes 10 time units.
Project Management Terms

<table>
<thead>
<tr>
<th>Activity</th>
<th>The process of doing work. The expenditure of a resource.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event</td>
<td>The accomplishment of work. A milestone.</td>
</tr>
<tr>
<td>Network</td>
<td>A graphical representation of an ordered series of activities and events.</td>
</tr>
<tr>
<td>Node</td>
<td>The place where an event connects with an activity in a network diagram.</td>
</tr>
<tr>
<td>Source Event</td>
<td>The beginning of a project.</td>
</tr>
<tr>
<td>Terminal Event</td>
<td>The end of a project.</td>
</tr>
<tr>
<td>Predecessor</td>
<td>The event that marks the beginning of an activity.</td>
</tr>
<tr>
<td>Successor</td>
<td>The event that marks the end of an activity.</td>
</tr>
</tbody>
</table>

CPM Notes

1. A network consists of interrelated and interdependent activities and events.
2. An activity refers to the actual process of doing work. It involves the expenditure of time, effort, money, and other resources. In networks, a line with an arrow at one end represents an activity.
3. An event refers to a specific and identifiable milestone, or accomplishment of work, at a specified time. Events only signify an expenditure of time, effort, money, or other resources. In networks, a circle or a small square represents an event.
4. A node is the place where an activity connects to an event.
5. By convention, the network flows from left to right. That is, time and progress of the project flow from left to right.
6. A network is constructed in such a fashion that planning and technological requirements are translated into precedence relationships. For example, in the construction of a building, the foundation laying activity must precede wall raising and roofing activities.
7. An activity is always bounded by two events called the start event and the end event. The start (or predecessor event) signifies the beginning of an activity, while the end (or successor event) marks the end of an activity.
8. The event that marks the beginning of the entire project is called the source event, while the event that marks the completion of the entire project is called the terminal event.
9. No activity can start until its start event is complete.
10. No event is complete until all activities leading to that event are complete.
11. A corollary of observations 9 and 10 is that loops or cycles are not permitted in networks.
12. In order to incorporate technological or managerial requirements, it is sometimes necessary to insert a dummy activity that does not require any time, effort, or resources for its completion.
13. A network path consists of a set of activities that connect the source event to the terminal event. Any non-trivial network will have more than one path. The longest path (in terms of time) is called the critical path. Paths other than the critical path are called non-critical paths or slack paths.
Is this what happened during your last project?

Project assigned
I’ve got an idea. Let’s start building our device.

Midpoint of project
The parts are back ordered. What do we do now?

One week before due date
I thought you were going to develop the test plans.

Project deadline
We’re finished! Too bad it doesn’t work very well...

If this sounds familiar, then...

YOU NEED TO KNOW CPM!

Determining the Critical Path

Once the network has been drawn, you are ready to find the critical path. The calculations required for determining the critical path are outlined in a PowerPoint presentation. Simply put, CPM stands for Critical Path Method used in planning and controlling time. It helps estimate time for project completion, pin-points activities critical for achieving the target. Above all, it gives a graphic view of the activities of a project which is easy to understand in one go.

BACKGROUND INFORMATION

As per information provided by J. Rashid, the project required basic facilities like land, building and machinery which have been shown in Table #1 along with estimated time. As would be observed that after acquiring land, the company can go for three activities: (i) placing order for machinery, (ii) construction of building, (iii) placing order for plastic bottles for water filling.

When buildings are complete, machines can installed and the same time, racks can be erected for storing plastic bottles. Thus the company can go for trials runs as necessary pre-requisites are in order.
Format of a network or CPM varies. Basic structure is given in the textbooks or Software (MS Project 2007) which can be suitably modified. Personally, I like a detailed format for presentation.

Here each box or node has nine cells as shown in its legend. ES means Early Start while EF mean Early Finish.

In sketch # 1, some information has already been inserted about a particular activity such as ID, Description & Duration. The activities are also placed keeping their relationships in view.

**Sketch # 1**
FORWARD PASS
Let us start with ES & EF. This is simple except where we confront a Merge Event. We would insert Zero (or expected date) in the first cell of activity #1. Since duration for this activity is 1 month, the EF would be 0 + 1 = 1. Same would be carried for activities #2, #3 & #4 and EF worked out accordingly.

The ES of activity #5 would be 9, same as EF of its predecessor #3. It is natural; you cannot start erecting racks (#5) unless buildings (#3) are complete.

When you go next to activity #6, you would see two activities (#2 & #3) are merging. Obviously, you cannot start installation of machines (#6) until the machinery has arrived as per order (#2) and buildings (#3) are complete. Here we would take maximum of EF of the both as both must be complete before #6 can be started. In the same way, we can complete rest of the diagram. We find that the project would take 19 months to be completed as shown in Sketch #2 below:

Sketch #2

BACKWARD PASS
As the name implies, we move backward to the origin. In this example, the project completion time is 19 months. So its Early Finish (EF) or Late Finish (LF) must be the same. In other words, call it early or late, it must be finished by 19th month.

We start from last activity #8 and insert 19 in its LF cell. This activity has duration of 4 months. Calculating backward, we find that its LS should be 15th months (19 - 4 = 15). In other, if #8 was started as late as on 15th months and it takes 4 months, it could still meet the deadline of 19 months.

We go back further and trace tails of any arrows touching #8. We find two, one linking with #7 (Store Raw Materials) and other with #6 (install machinery). We would put 15 in the LF Cells of #6 & #7. We can now calculate their LS by deducting from 15 their respective durations. For example, the LS of #6 should now be 9 (15-6=9).
We can continue likewise till we hit a "Burst Event" like #3 & #1. For explanation, #1 would be better as three activities are leading back to this activity. (There are three tails touching #1.) In such a situation, we shall take minimum of the LS of the three which is one month and it would be LF of #1. Why? The longest activity after #1 is #3. In order to enable #3 to start on time, it is essential that the preceding activity is completed well in time.

The complete working is shown in Sketch # 3 below:

**Sketch # 3**

![Network Diagram]

- **Project Management - Project Evaluation and Review Technique**
  Project Evaluation and Review Technique - Complex projects will have a significant number of activities that need to be performed, both in sequence and in parallel.

### 1. Exercise 1

<table>
<thead>
<tr>
<th>ACTIVITY</th>
<th>ESTIMATED HOURS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A –——&gt; B</td>
<td>receive assignment</td>
</tr>
<tr>
<td>A –——&gt; D</td>
<td>scan text</td>
</tr>
<tr>
<td>A –——&gt; E</td>
<td>study text</td>
</tr>
<tr>
<td>B –——&gt; C</td>
<td>form study group</td>
</tr>
<tr>
<td>B –——&gt; D</td>
<td>do homework</td>
</tr>
<tr>
<td>C –——&gt; F</td>
<td>study with group</td>
</tr>
<tr>
<td>D –——&gt; F</td>
<td>correct homework</td>
</tr>
<tr>
<td>E –——&gt; F</td>
<td>outline text</td>
</tr>
</tbody>
</table>
2. Exercise 2

Name the critical path.

Exercise 2 Solution: The critical path is A-B-C-F-G.

**Assessment**

1. In network theory, the accomplishment of work is called
   a) an event
   b) a node
   c) an activity
   d) a predecessor
2. In network theory, the process of doing work is called
   a) an event
   b) a node
   c) an activity
   d) a predecessor

3. In network theory, a circle represents
   a) an event
   b) a node
   c) an activity
   d) a predecessor

4. In the network above, which of the following is not a valid path?
   a) A--B--D--E--F
   b) A--C--B--E--F
   c) A--B--C--E--F
   d) A--B--D--F

5. When is the earliest event C can be accomplished?
   a) 2 time units
   b) 4 time units
   c) 5 time units
   d) 6 time units
Practice Problem: Project Management

**Problem 1:**

The following represent activities in a major construction project. Draw the network to represent this project.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Immediate Predecessor</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-</td>
</tr>
<tr>
<td>B</td>
<td>-</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td>D</td>
<td>B</td>
</tr>
<tr>
<td>E</td>
<td>B</td>
</tr>
<tr>
<td>F</td>
<td>C, E</td>
</tr>
<tr>
<td>G</td>
<td>D</td>
</tr>
<tr>
<td>H</td>
<td>F, G</td>
</tr>
</tbody>
</table>

**Problem 2:**

Given the following Time Chart and Network Diagram, find the Critical Path.

<table>
<thead>
<tr>
<th>Activity</th>
<th>a</th>
<th>m</th>
<th>b</th>
<th>t</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>1/9</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1/9</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>5</td>
<td>12</td>
<td>6</td>
<td>16/9</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>4/9</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1/9</td>
</tr>
</tbody>
</table>
Problem 3:
What is the variance in completion time for the critical path found in Problem 2?

Problem 4:
A project has an expected completion time of 40 weeks and a standard deviation of 5 weeks. It is assumed that the project completion time is normally distributed.
(a) What is the probability of finishing the project in 50 weeks or less?
(b) What is the probability of finishing the project in 38 weeks or less?
(c) The due date for the project is set so that there is a 90% chance that the project will be finished by this date. What is the date?

Problem 5:
Development of a new deluxe version of a particular software product is being considered. The activities necessary for the completion of this project are listed in the table below along with their costs and completion times in weeks.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Normal Time</th>
<th>Crash Time</th>
<th>Normal Cost</th>
<th>Crash Cost</th>
<th>Immediate Predecessor</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>3</td>
<td>2,000</td>
<td>2,600</td>
<td>-</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>1</td>
<td>2,200</td>
<td>2,800</td>
<td>A</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>3</td>
<td>500</td>
<td>500</td>
<td>A</td>
</tr>
<tr>
<td>D</td>
<td>8</td>
<td>4</td>
<td>2,300</td>
<td>2,600</td>
<td>A</td>
</tr>
<tr>
<td>E</td>
<td>6</td>
<td>3</td>
<td>900</td>
<td>1,200</td>
<td>B, D</td>
</tr>
<tr>
<td>F</td>
<td>3</td>
<td>2</td>
<td>3,000</td>
<td>4,200</td>
<td>C, E</td>
</tr>
<tr>
<td>G</td>
<td>4</td>
<td>2</td>
<td>1,400</td>
<td>2,000</td>
<td>F</td>
</tr>
</tbody>
</table>

(a) What is the project expected completion date?
(b) What is the total cost required for completing this project on normal time?
(c) If you wish to reduce the time required to complete this project by 1 week, which activity should be crashed, and how much will this increase the total cost?
**ANSWERS:**

**Problem 1:**

![Diagram](image)

**Problem 2:**

Critical path: ACDE = 14

![Diagram](image)

**Problem 3:**

Total variance = $\sum$ variances of activities on critical path

Total variance = $1/9 + 16/9 + 4/9 + 1/9 = 2/2/9 = 2.55$

And $\sigma = \sqrt{2.55} = 1.6$
Problem 4:

(a) \[ Z = \frac{X - \mu}{\sigma} = \frac{50 - 40}{5} = 2 \]

Therefore: \[ P(X \leq 50) = P(Z \leq 2) = 0.97725 \]

(b) \[ Z = \frac{X - \mu}{\sigma} = \frac{-2}{5} = -0.4 \]

Therefore: \[ P(X \leq 38) = P(Z \leq -0.4) = 0.34458 \]

(c) \[ 90\% \geq Z = 1.28 = (\chi - \mu)/\sigma = \chi - 40/5 \]

Therefore: \[ \chi = 1.28*5 + 40 = 46.4 \text{weeks} \]

Problem 5:

(a)

Project completion time is therefore \[ t_A + t_D + t_E + t_F + t_G = 4 + 8 + 6 + 3 + 4 = 25 \]

(b) Total cost = $2,000 + $2200 + $500 + $2,300 + $900 + $3,000 + $1,400 = $12,300

(c) Crash D 1 week at an additional cost of \[ \frac{\$2,600 - \$2,300}{8 - 4} = \frac{\$300}{4} = $75 \]
Practice Problems: Waiting-Line Models

Problem 1:
A new shopping mall is considering setting up an information desk manned by one employee. Based upon information obtained from similar information desks, it is believed that people will arrive at the desk at a rate of 20 per hour. It takes an average of 2 minutes to answer a question. It is assumed that the arrivals follow a Poisson distribution and answer times are exponentially distributed.

(a) Find the probability that the employee is idle.
(b) Find the proportion of the time that the employee is busy.
(c) Find the average number of people receiving and waiting to receive some information.
(d) Find the average number of people waiting in line to get some information.
(e) Find the expected time a person seeking information spends in the system.
(f) Find the expected time a person spends just waiting in line to have a question answered (time in the queue).

Problem 2:
Assume that the information desk employee in Problem 1 earns $10 per hour. The cost of waiting time, in terms of customer unhappiness with the mall, is $12 per hour of time spent waiting in line. Find the total expected costs over an 8-hour day.

Problem 3:
The shopping mall has decided to investigate the use of two employees on the information desk.

(a) Find the probability of no people in the system.
(b) Find the average number of people waiting in this system.
(c) Find the expected time a person spends waiting in this system.
(d) Assuming the same salary level and waiting costs as in Problem 2, find the total expected costs over an 8-hour day.

Problem 4:
Three students arrive per minute at a coffee machine that dispenses exactly four cups per minute at a constant rate. Describe the system parameters.

Problem 5:
A repairman at a local metal working shop services their five drill presses. Service time averages 10 minutes and is exponentially distributed. Machines breakdown after an average of 70 minutes operation (following a Poisson distribution). Describe the major system characteristics.
ANSWERS

Problem 1:

(a) \( P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{20}{30} = 0.33 \Rightarrow 33\% \)

(b) \( p = \frac{\lambda}{\mu} = 0.66 \)

(c) \( L_s = \frac{\lambda}{\lambda - \mu} = \frac{20}{30 - 20} = 2 \text{ people} \)

(d) \( L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{20^2}{30(30 - 20)} = 1.33 \text{ people} \)

(e) \( W_s = \frac{1}{\lambda - \mu} = \frac{1}{30 - 20} = 0.10 \text{ hours} \)

(f) \( W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{20}{30(30 - 20)} = 0.0667 \text{ hours} \)

Problem 2:

From the solution to Problem 1:

The average person waits 0.0667 hours and there are 160 (20 arrivals * 8 hours) arrivals per day.

Therefore: Total waiting time = 160 \times 0.0667 = 10.67 hours

Total cost for waiting = Total waiting time \times Cost per hour = 10.67 \times $12 = $128 per day.

Salary cost = 8 hours \times $10 = $80

Total cost = Salary cost + Waiting cost = $80 + $128 = $208 per day.

Problem 3:

\( \lambda = 20 \text{ per hour} \quad \mu = 30 \text{ per hour} \quad M = 2 \text{ open channels (servers)} \)

(a)

\[
P_0 = \frac{1}{1 \left( \frac{20}{30} \right)^0 + \frac{1}{1!} \left( \frac{20}{30} \right)^1 + \frac{1}{2} \left( \frac{20}{30} \right)^2 \left( \frac{2(30)}{30(30 - 20)} \right)}
\]
\[
\frac{1}{1 + \frac{2}{3} + \frac{1}{3}} = \frac{1}{2} = 50\%
\]

(b) \[L_s = \left( \frac{(20)(30)\left(\frac{20}{30}\right)^2}{1[(2)(30) - 20]^2} \right) \left(\frac{1}{2}\right) + \frac{20}{30} \]

\[= \left[ \frac{800}{3} \right] \frac{1}{2} + \frac{2}{3} \]

\[= \frac{1}{12} + \frac{8}{12} = 0.75 \text{ people} \]

(c) \[W_s = \frac{L_s}{\lambda} = \frac{0.75}{20} = 0.0375 \text{ hours} \]

Problem 4:

\[L_q = \frac{\lambda^2}{2 \mu (\mu - \lambda)} = 1.125 \text{ people in the queue on average} \]

\[W_q = \frac{\lambda}{2 \mu (\mu - \lambda)} = 0.375 \text{ minutes in the queue waiting} \]

\[L_s = L_q + \frac{\lambda}{\mu} = 1.87 \text{ people in the system} \]

\[W_s = W_q + \frac{1}{\mu} = 0.625 \text{ minutes in the system} \]
Problem 5:

\( N = 5 \)
\( T = 10 \) minutes
\( U = 70 \) minutes
\( M = 1 \) server

\[ X = \frac{T}{T+U} = \frac{10}{10+70} = 0.125 \]

From Table D.8: where \( X = .125 \) and number of service channels = 1, \( D = 0.473 \), \( F = 0.920 \)

Average number waiting = \( L = N(1 – F) = 5(1 – 0.920) = 0.4 \)

Average number of machines running = \( J = NF(1 – X) = 5(0.920)(1 – 0.125) = 4.025 \) machines

Average number of machines being serviced = \( H = FNX = (0.920)(5)(0.125) = 0.575 \) machines

Probability of no wait = \( 1 – D = 1 – 0.473 = 0.527 \)
Practice Problems: Transportation Models

Problem 1:

John Galt Shipping wishes to ship a product that is made at two different factories to three different warehouses. They produce 18 units at Factory A and 22 units at Factory B. They need 10 units in warehouse #1, 20 units in warehouse #2, and 10 units in warehouse #3. Per unit transportation costs are shown in the table below. How many units should be shipped from each factory to each warehouse?

<table>
<thead>
<tr>
<th></th>
<th>Warehouse #1</th>
<th>Warehouse #2</th>
<th>Warehouse #3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant A</td>
<td>$4</td>
<td>$2</td>
<td>$3</td>
</tr>
<tr>
<td>Plant B</td>
<td>$3</td>
<td>$2</td>
<td>$1</td>
</tr>
</tbody>
</table>

Problem 2:

Assume that in Problem 1 the demand at each warehouse is increased by 4 units. Now how many units should be shipped from each factory to each warehouse?
ANSWERS

Problem 1:

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>Warehouses</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>#1</td>
<td>#2</td>
</tr>
<tr>
<td>Factory A</td>
<td></td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>Factory B</td>
<td></td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>

Problem 2:

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>Warehouses</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>#1</td>
<td>#2</td>
</tr>
<tr>
<td>Factory A</td>
<td></td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>Factory B</td>
<td></td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>Dummy Factory</td>
<td></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>14</td>
<td>24</td>
</tr>
</tbody>
</table>

Note that we had to use a “Dummy Factory” to supply the extra products as demand exceeds the quantity available from just the two factories.